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MATHEMATICS

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Mathematical Reviews

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PROBABILITY

See also A10935, A11094, 11441, 11456, 11504, 11506, 11542, 11999.

11416:

Doob, J. L.; Snell, J. L.; Williamson, R. E. Application of boundary theory to sums of independent random variables. Contributions to probability and statistics, pp. 182–197. Stanford Univ. Press, Stanford, Calif., 1960.

Specializing the boundary theory developed by Doob for discrete-time Markov chains [J. Math. Mech. **8** (1959), 433–458, 993; MR **21** #5825] to the case of sums of independent and identically distributed random variables, the authors determine the minimal functions (Martin boundary) for an N-dimensional lattice random walk (with illustrations); characterize the approach to the boundary for the (one-dimensional) space-time random walk in terms of the generating function of summands; and obtain a "relativized Fatou theorem" regarding the a.e. nontangential limit of the ratio of positive regular functions for the latter random walk.

J. Kiefer (Ithaca, N.Y.)

11417:

Ghurye, S. G. Characterization of some location and scale parameter families of distributions. Contributions to probability and statistics, pp. 203–215. Stanford Univ. Press, Stanford, Calif., 1960.

Various distributions such as the normal, exponential, and uniform are characterized by the uniform conditional distribution of a sufficiently large number of independent, identically distributed random variables on an appropriate surface, generalizing the result of Zinger [Vestnik Leningrad. Univ. **11** (1956), no. 1, 53–56; MR **17**, 863]. For the normal case similar characterizations of the Wishart and multivariate normal distributions are given.

J. Kiefer (Ithaca, N.Y.)

11418:

Marshall, Albert W.; Olkin, Ingram. A bivariate Chebyshev inequality for symmetric convex polygons. Contributions to probability and statistics, pp. 299–308. Stanford Univ. Press, Stanford, Calif., 1960.

Let a symmetric open plane polygonal region T be given as the intersection of strips $S_1, \cdots, S_m$, and let X be a random 2-vector with $EXX' = \Sigma$. The authors show that the upper bound $\min\{\text{tr } A\Sigma;\ x'Ax \geq 1,\ x \notin T\}$ on $P\{X \notin T\}$ can be obtained as $\min\{\text{tr } B\Sigma;\ x'Bx \geq 1,\ x \notin S_i \cap S_j\}$ for some i, j, if $\{x: x'Bx < 1\} \in T$ for some such B. Otherwise, the minimum of the numbers achieved by replacing $S_i \cap S_j$ by the hexagon $S_i \cap S_j \cap S_k$, yields the bound. Sharpness is proved if $m = 2$ and $m = 3$.

J. Kiefer (Ithaca, N.Y.)

11419:

Smith, Walter L. Infinitesimal renewal processes. Contributions to probability and statistics, pp. 396–413. Stanford Univ. Press, Stanford, Calif., 1960.

Let $X(t)$ have stationary increments with $X(0) = 0$, $EX^r(1) = m_r$, $F(x, t) = \Pr\{X(t) \leq x\}$, and $F(x) = \int_0^\infty e^{-t} F(x, t)dt$, $F_k(x) = F_1(x) * F_{k-1}(x)$. Theorem 1: If $m_1 > 0$, the expected length of time that $X(t) \leq x$ is $H(x) = \sum_1^\infty F_k(x)$. Hence, Blackwell's theorem [Pacific J. Math. **3** (1953), 315–320; MR **14**, 994] applies, and also Theorem 2: If $m_2 < \infty$, then $H(x) \to x/m_1 + (m_2 - m_1)^2/2m_1^2 + o(1)$ as $x \to \infty$ (which is new also for the discrete time general renewal process). Necessary conditions and sufficient conditions are given for a renewal density theorem (concerning the derivative of H).

J. Kiefer (Ithaca, N.Y.)

11420:

Wolfowitz, J. Convergence of the empiric distribution function on half-spaces. Contributions to probability and statistics, pp. 504–507. Stanford Univ. Press, Stanford, Calif., 1960.

Let $X_1, X_2, \cdots$ be independent, identically distributed chance vectors, and for any constant vector p let F_p and F_{pn} be the d.f. of the real random variable $p'X_1$ and the empiric d.f. of $p'X_1, \cdots, p'X_n$, respectively. The author proves that

$$P\{\lim_n \sup_p \sup_x |F_p(x) - F_{pn}(x)| = 0\} = 1,$$

improving on previous results (proved under restrictions on the d.f. of X_1) of the author [Ann. Math. Statist. **25** (1954), 131–138; MR **16**, 808], Blum [ibid. **26** (1955), 527–529; MR **17**, 48], and Fortet and Mourier [Ann. Sci. École Norm. Sup. (3) **70** (1953), 267–285; MR **15**, 993].

J. Kiefer (Ithaca, N.Y.)

11421:

Newman, Donald J.; Shepp, Lawrence. The double dixie cup problem. Amer. Math. Monthly **67** (1960), 58–61.

Assume that a set has n distinct types of objects available in equal proportions. The authors give an asymptotic formula for the expectation of the number of independent trials required to obtain m complete sets of n distinct objects; generating functions are used. This is also known as the "collector's problem".

R. F. Tate (Seattle, Wash.)

11422:

Morgenthaler, George W. On Bernoulli play with limited resources. Amer. Math. Monthly **67** (1960), 344–348.

Let $\{x_k\}$ be a sequence of mutually independent Bernoulli random variables. Suppose that at each trial a non-negative quantity of a limited resource is forfeited, say σ_1, if the trial outcome is zero, and $\sigma_2 > \sigma_1$, if the outcome is 1. Every play will then terminate after a finite number of trials, N. The author derives simple exact expressions for the probabilities $P(j) = \Pr\{x_1 + \cdots + x_N = j\}$, $Q(N) = \Pr\{\text{number of trials in a play} = N\}$, under the assumption that trials are continued until depletion of the resource. Several applications are indicated.

E. Parzen (Stanford, Calif.)

11423:

Katti, S. K. Moments of the absolute difference and absolute deviation of discrete distributions. Ann. Math. Statist. **31** (1960), 78–85.

The author discusses the evaluation of $\Delta_r = E|X_1 - X_2|^r$, where X_1 and X_2 are two independent random variables with distributions—possibly different—within one of the following families of distributions: Poisson, Pascal, and Binomial. Methods are also given to evaluate Δ_r when X_2 is a fixed constant and when X_1 is distributed as a Poisson, a Pascal, a Binomial, a Hypergeometric and a Logarithmic random variable.

Hian Liang Ang (Bandung)

11424:

Bahadur, R. R. Some approximations to the binomial distribution function. Ann. Math. Statist. **31** (1960), 43–54.

Let $0 < p < 1$, $q = 1 - p$, $b(n, k) = \binom{n}{k} p^k q^{n-k}$, $B_n(k) = \sum_j^n b(n, j)$, $np \leq k \leq n$. It is first shown that $B_n(k) = qb(n, k)\, {}_2F_1(n+1, 1; k+1; p)$. This representation is exploited to give bounds for $B_n(k)$ and some asymptotic estimates. Define $x = (k - np)(npq)^{-1/2}$, $z = (n+1)p(k+1)^{-1}$, $A_n(k) = q\, b(n, k)(1-z)^{-1}$; then $1 \leq A_n(k)/B_n(k) \leq 1 + x^{-2}$. Let $N_n(k)$ denote the normal approximation to $B_n(k)$, $C_n(k) = (x + [q/np]^{1/2})\ (2\pi)^{1/2} \exp(x^2/2)$. If k varies with n so that $x \geq 0$ for each n, then $(A_n N_n C_n)/B_n \to 1$ as $n \to \infty$. As $n \to \infty$, the equivalence of (i) $A_n/B_n \to 1$, (ii) $B_n \to 0$, (iii) $x_n \to \infty$, and of (a) $N_n/B_n \to 1$, (b) $A_n C_n \to 1$, is proved. The latter equivalence is further investigated for certain values of p.

I. Olkin (Minneapolis, Minn.)

11425:

Wendel, J. G. The non-absolute convergence of Gil-Pelaez' inversion integral. Ann. Math. Statist. **32** (1961), 338–339.

An inversion formula for a distribution function $F(x)$ in terms of its characteristic function $\varphi(t)$ is

$$F(x) = \tfrac{1}{2} - \pi^{-1} \int_{+0}^{+\infty} \operatorname{Im}\{e^{-itx}\varphi(t)/t\}\, dt$$

where the arrows signify that the integral might be improper at either or both limits. The author emphasizes that the lower limit is usually improper, and shows that the set of distributions $F(x)$ for which the inversion formula for $F(0)$ is proper at the lower limit is a set of the first category in the space of distributions metrized by $\rho(F, G) = \text{total variation of } F(x) - G(x)$. {Reviewer's remark: Although the above inversion formula is often attributed to Gil-Pelaez [Biometrika **38** (1951), 481–482; MR **13**, 666], a similar formula was given earlier by Gurlund [Ann. Math. Statist. **19** (1948), 228–237; MR **9**, 582].}

E. Parzen (Stanford, Calif.)

11426:

Teicher, Henry. Identifiability of mixtures. Ann. Math. Statist. **32** (1961), 244–248.

If $F(x; \alpha)$ is a family of cumulative distribution functions with a parameter α, and $H(x) = \int F(x; \alpha) dG(\alpha)$, when can the cumulative distribution function $G(\alpha)$ be uniquely determined from $H(x)$ and $F(x; \alpha)$? The author's main results are that this is possible if α is a scalar and (i) $F(x; \alpha)$ is such that the convolution of $F(x; \alpha)$ and $F(x; \beta)$ is $F(x; \alpha + \beta)$, where α ranges over either non-negative integers, or non-negative rationals, or non-negative real numbers, or (ii) if α is a translation parameter, unless the Fourier transform of $F(x; \alpha)$ is identically zero in some non-degenerate real interval, or (iii) if $F(0+; \alpha) = 0$ and α is a scalar parameter, unless the Fourier transform of $F(e^x; \alpha)$ is identically zero in some non-degenerate real interval.

E. M. L. Beale (London)

11427:

Feldman, Jacob. Examples of non-Gaussian quasi-invariant distributions in Hilbert space. Trans. Amer. Math. Soc. **99** (1961), 342–349.

Associated with a representation of the so-called canonical commutation relations (as well as with certain stochastic processes) is a probability distribution in an infinite-dimensional linear space, which is quasi-invariant, i.e., absolutely continuous with respect to its translates. Conversely, for any quasi-invariant distribution there is a collection of unitary equivalence classes of representations of the canonical commutation relations, two distributions giving the same collection if and only if they are mutually absolutely continuous. In the case of a finite-dimensional space it is classical that there is a unique quasi-invariant distribution, within absolute continuity, a result related to the Stone-von Neumann theorem asserting the existence of a unique irreducible representation of the canonical commutation relations in this case.

The problem of classifying the representations of the canonical commutation relations for a quantum field thereby gives rise to that of classifying the equivalence classes under mutual absolute continuity of quasi-invariant distributions on Hilbert space. A simple hypothesis consistent with the known results was that every such equivalence class contained a Gaussian (normal) distribution, at least if it was ergodic (i.e., admitted no non-trivial invariant measurable subset under translations). The author constructs examples of ergodic quasi-invariant distributions on Hilbert space not equivalent to any Gaussian distributions. The proof of non-equivalence necessarily involves a substantial element of novelty.

I. E. Segal (Cambridge, Mass.)

11428:

Ibragimov, I. A. On asymptotic distribution of values of certain sums. Vestnik Leningrad. Univ. **15** (1960), no. 1, 55–69. (Russian. English summary)

Let $f(t)$ be periodic with period 1, $f \in L_2(0, 1)$, $\int_0^1 f(t)dt = 0$. Let $[f]_k(t) = 2^k \int_{A_{jk}} f(t)dt$, $j = 1, 2, \cdots, 2^k$, where $A_{jk} =$

$[(j-1)2^{-k}, j\cdot 2^{-k}]$. Theorem 1.1: If $\sum_{k=1}^{\infty} \|f-[f]_k\| < \infty$, then

$$\|f\|^2 + 2\sum_{k=1}^{\infty} \int_0^1 f(t)f(2^k t)\,dt = \sigma^2 < \infty,$$

and if $\sigma \neq 0$, then the Lebesgue measure of the set $\{t \mid 0<t<1,\ \sum_{k=0}^{n-1} f(2^k t) < z\sigma n^{1/2}\}$ converges to

$$\Phi(z) = (2\pi)^{-1/2} \int_{-\infty}^{z} \exp(-u^2/2)\,du.$$

Theorem 2.1 is an extension of Theorem 1.1 to complex-valued functions. For $0<t<1$ let the positive integers $a_i(t)$ be defined by the continued fraction

$$t = 1/[a_1(t) + 1/(a_2(t)\cdots)] = [a_1(t), a_2(t), \cdots],$$

and let $Tt = [a_2(t), a_3(t), \cdots]$, $\mu(A) = \int_A (1+t)^{-1}dt/\log 2$. Theorem 1.2 is a close analogue of Theorem 1.1 for the μ measure of the set $\{t \mid \sum_{k=0}^{n-1} f(T^k t) < zn^{1/2}\}$. These theorems are deduced from a central limit theorem for sums of terms $f_k = f(x_k, x_{k+1}, \cdots)$, where $\{x_k\}$ is a stationary sequence of random variables which satisfies a certain mixing condition (Theorem 1); the proof of the latter is to appear elsewhere. An alternative proof of Theorem 1.1 is given by a method similar to that used by M. Kac [Ann. of Math. (2) **47** (1946), 33–49; MR **7**, 436] to prove the same result under stronger assumptions. For a special case of Theorem 1.2 see W. Doeblin, Compositio Math. **7** (1940), 353–371 [MR **2**, 107].

W. Hoeffding (Chapel Hill, N.C.)

11429:

Burke, C.; Rosenblatt, M. Consolidation of probability matrices. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 7–8. (French summary)

The authors present a brief summary of results which appeared in another paper [Ann. Math. Statist. **29** (1958), 1112–1122; MR **21** #367]. *J. L. Snell* (Hanover, N.H.)

11430:

Fuchs, Aimé. Temps d'atteinte des groupes finaux dans les chaînes de Markov. C. R. Acad. Sci. Paris **250** (1960), 281–283.

The author considers a finite Markov chain. Let $\mathscr{E}$ be the set of transient series, assumed to be not empty. Let $\mathscr{F}$ be the set of non-transient states. Let $M_{i\mathscr{F}}$ for i in $\mathscr{E}$ be the mean time starting at i to enter $\mathscr{F}$ for the first time. The principle result states is that $M_{i\mathscr{F}} = 1 + \sum_{n=1}^{\infty} P_{it}^n$ where $P_{it}^n = \sum_{j\in t} P_{ij}^n$. *J. L. Snell* (Hanover, N.H.)

11431:

Watanabe, Takesi. On the theory of Martin boundaries induced by countable Markov processes. Mem. Coll. Sci. Univ. Kyoto Ser. A Math. **33** (1960/61), 39–108.

The paper deals with certain types of countable Markov processes and the corresponding potential theory. Let X be a countable set, $\bar{X}$ the same set with ∞ adjoined as an isolated point, T the non-negative reals (or the non-negative integers) with $+\infty$ adjoined. A path is a function w: $T \to \bar{X}$, its value at time t is $x_t(w)$, and $\sigma_\infty(w)$ is $\inf\{t;\ x_t(w) = \infty\}$; the space W of paths comprises all w satisfying (1) $x_{+\infty}(w) = \infty$, (2) $x_t(w) = \infty$ for every $t \geqq \sigma_\infty(w)$, (3) $x_t(w)$ is continuous on the right for every t and has only discontinuities of the first kind for $t < \sigma_\infty(w)$. The discussion is confined to Markov processes on $\bar{X}$ which have stationary transitions and for which W may be taken to be the space of sample paths. Chapter 1 treats basic matters like the strong Markov property, the decomposition of $\bar{X}$ into irreducible recurrent sets and the set of nonrecurrent states, the construction of such processes. Chapters 2, 3 and 4 develop in detail the theory of superharmonic and excessive functions relative to a fixed process; in particular, these chapters establish properties of the Martin completion of X (including the canonical integral representation of positive superharmonic functions) that were stated without proof by Doob [J. Math. Mech. **8** (1959), 433–458; MR **21** #5825; especially § 8 and § 9]. Chapter 5 presents many examples illustrating the general theory and sketches the dual theory of superharmonic set functions. The paper extends to continuous-parameter processes many results that were known for discrete-parameter processes, it completes several points in the theory of Markov processes and the related superharmonic functions, and it provides an excellent systematic exposition of that theory. {In Theorem 3.4 (iii) the arrow should be reversed.} *G. A. Hunt* (Princeton, N.J.)

11432:

Watanabe, Takesi. A probabilistic method in Hausdorff moment problem and Laplace-Stieltjes transform. J. Math. Soc. Japan **12** (1960), 192–206.

In § 2 of this paper the author constructs explicitly the Martin boundary corresponding to the Markov process which has for state space the set of pairs (n, i) with $n \geqq i = 0, 1, 2, \cdots$, and which has for possible transitions $(n, i) \to (n+1, i)$ and $(n, i) \to (n+1, i+1)$, each with probability $\frac{1}{2}$. In § 3 he shows that general theorems concerning the Martin boundary (sketched in § 1, developed in detail in the paper [#11431] above) yield in this context the main theorems concerning the Hausdorff moment problem [Widder, *The Laplace transform*, Princeton Univ. Press, Princeton, N.J., 1941; MR **3**, 232; Theorems II 6.1, III 2b, III 4a]. In § 4 a similar construction gives the corresponding theorems for the Laplace-Stieltjes transform. *G. A. Hunt* (Princeton, N.J.)

11433:

Moy, Shu-Teh C. Asymptotic properties of derivatives of stationary measures. Pacific J. Math. **10** (1960), 1371–1383.

Let μ, ν be probability measures on the infinite product σ-algebra $\prod_n S_n$ of subsets of the product space $\prod_n X_n$, $X_n = X$, $S_n = S$, and S is a σ-algebra of subsets of the nonempty set X. Let ν be absolutely continuous with respect to μ. The paper is concerned with the asymptotic properties of the derivatives, assuming ν to be stationary and μ to be Markovian with stationary transitions. For the case where X is countable, the author obtains a generalization of the McMillan ergodic theorem of information theory. The proofs use ergodic theory and Doob's martingale convergence theorem.

A. V. Balakrishnan (Los Angeles, Calif.)

11434:

Pyke, Ronald. On centering infinitely divisible processes. Ann. Math. Statist. **31** (1960), 797–800.

This note characterizes centering functions preserving stationarity of increments for processes with stationary independent increments. *H. Teicher* (New York)

11435:

Debry, Robert. Introduction au problème des files d'attente. Bull. Soc. Math. Belg. **12** (1960), 28–56.

In this expository paper, the author illustrates queueing theory by giving transient and equilibrium solutions to the simple Poisson queue. Similarly he studies a preemptive priority queueing problem in which both arrival and service distributions satisfy Poisson assumptions. Finally he illustrates the imbedded Markov chain method by applying it to queues with bulk service along familiar lines followed by N. T. J. Bailey [J. Roy. Statist. Soc. Ser. B **16** (1954), 80–87; MR **16**, 148].

T. L. Saaty (Silver Springs, Md.)

11436:

Dhondt, André. Sur le comportement transitoire du processus d'attente simple. Cahiers Centre Études Rech. Oper. **2**, 223–233 (1960).

A generalization of earlier derivations of the transient behavior of a simple queue, with Poisson arrivals and exponential serving times, for the case when both mean arrival rate and mean service vary with time. In terms of the appropriate time variable, the integral of the service rate, the formulas come out the same as for the constant-rate formulas, derived earlier.

P. M. Morse (Cambridge, Mass.)

11437:

Miller, Rupert G., Jr. Priority queues. Ann. Math. Statist. **31** (1960), 86–103.

Steady-state solutions for single-server waiting lines are discussed, with different arrival and service statistics. Under various assumptions in this paper the following quantities have either been obtained explicitly or characterized as the unique (subject to regularity conditions) solution to a functional equation: the generating function for the stationary probabilities on the number of priority and non-priority items ($K=2$) in the queue, the Laplace-Stieltjes transforms of the waiting-time distributions, the Laplace-Stieltjes transform of the distribution of a busy period, and the generating function for the probabilities on the number of items serviced during a busy period. For most of the distributions mentioned the first two moments are computed.

P. M. Morse (Cambridge, Mass.)

11438:

Gumbel, Harold. Waiting lines with heterogeneous servers. Operations Res. **8** (1960), 504–511.

Author's summary: "A 'multiple-booth' server system with nonhomogeneous servers is analyzed under the assumption of Poisson distributed arrivals and exponential service time with different individual mean service rates for each server. Explicit expressions for the state probabilities are obtained in closed form under 'steady-state' conditions and the expected length of the waiting line is derived therefrom. It is shown that an 'equivalent' system with homogeneous servers does not exist. The error incurred in assigning each server the arithmetic mean of the service rates of all of them is analyzed and illustrated if expected number in the system is used as a criterion of comparison." *P. M. Morse* (Cambridge, Mass.)

STATISTICS

See also 11511, 12009.

11439:

Loveday, Robert. ★A second course in statistics. Cambridge University Press, New York, 1961. xi+155 pp. $1.85.

[Sequel to the author's *A first course in statistics*, Cambridge Univ. Press, New York, 1958.] Textbook in statistics for use in sixth-forms of British schools.

P. H. Diananda (Singapore)

11440:

Alexander, Howard W. ★Elements of mathematical statistics. A Wiley Publication in Mathematical Statistics. John Wiley & Sons, Inc., New York-London, 1961. xi+367 pp. $7.95.

This book is intended to provide the basis for a one-year first course in mathematical statistics with the only prerequisite being a first-year calculus course. (It is designed also to enable a reasonable one-semester course to be taught from it.) The first three chapters (slightly more than half the book) are entitled Discrete distributions, Continuous distributions and Sampling distributions I, respectively. The exposition in these is clear and well-motivated. None of the development depends upon generating functions, although supplementary material (and alternative proofs of results) and exercises on them are included. The fourth chapter (of 35 pages) is entitled Statistical inference. It consists mainly of an introduction to tests of significance. (There is a section on the computation of the power of certain tests, but the utility and importance of this side of hypothesis testing never comes through.) There is a brief treatment of confidence intervals and a four-page section on point estimation which is necessarily sketchy, and rather muddy in its definitions. Chapter 5—Sampling distributions II—develops the Poisson, t, F, etc. The treatment is again clear and tied to the intuition of the student. (Unfortunately the student is taught a false rule for the calculation of degrees of freedom in goodness-of-fit tests where parameters have been estimated from the data.) Consistent with the author's aim for a mathematically elementary book which proves things is his (very well-written) moment proof of the central limit theorem with which he closes the chapter. The last two chapters, embracing 45 pages, give good skeleton treatments of regression and correlation, and of the analysis of variance.

The book has many exercises (answers to the odd-numbered are given) and worked examples. The make-up of the book is good as to typography, figures, tables, index, etc. Misprints are few. {The only serious one noted was the omission of the factor $N/(N-1)$ from all probabilities in exercise 27.6.} The book's virtues are lucidity and appeal to the student's intuition. Its main shortcoming is the inadequacy of its treatment of inference, which is not only very brief but conceptually meagre.

L. E. Moses (Stanford, Calif.)

11441:

★Contributions to probability and statistics: Essays in honor of Harold Hotelling. Edited by Ingram Olkin, Sudhish G. Ghurye, Wassily Hoeffding, William G. Madow, Henry B. Mann. Stanford Studies in Mathematics and Statistics, 2. Stanford University Press, Stanford, Calif., 1960. x+517 pp. $6.50.

The volume contains 41 essays, together with a bibliography of Hotelling's works; those papers not being reviewed in this section are as follows: #11416–11420, 11508–11509, 11979, 11996, 11999.

11442:

Madow, William G. Harold Hotelling. Contributions to probability and statistics, pp. 3–5. Stanford Univ. Press, Stanford, Calif., 1960.

A short scientific biography. *J. Kiefer* (Ithaca, N.Y.)

11443:

Neyman, Jerzy. Harold Hotelling—a leader in mathematical statistics. Contributions to probability and statistics, pp. 6–10. Stanford Univ. Press, Stanford, Calif., 1960.

A discussion of Hotelling's influence, and especially of the effect of his 1940 paper on the teaching of statistics [see following review]. *J. Kiefer* (Ithaca, N.Y.)

11444:

Hotelling Harold. The teaching of statistics. Contributions to probability and statistics, pp. 11–24. Stanford Univ. Press, Stanford, Calif., 1960.

A reprinting of the paper first published in Ann. Math. Statist. **11** (1940), 457–470. *J. Kiefer* (Ithaca, N.Y.)

11445:

Anderson, R. L. Some remarks on the design and analysis of factorial experiments. Contributions to probability and statistics, pp. 35–56. Standford Univ. Press, Standford, Calif., 1960.

A survey article, including several illustrative examples and a suggestion for choosing linear parametric functions in a manner which will "more nearly coincide with the sequential nature of [the author's approach to] examining universal results". *J. Kiefer* (Ithaca, N.Y.)

11446:

Anderson, T. W.; Friedman, Milton. A limitation of the optimum property of the sequential probability ratio test. Contributions to probability and statistics, pp. 57–69. Stanford Univ. Press, Stanford, Calif., 1960.

The curtailed single-sampling acceptance plan S based on Bernoulli trials and which stops after the first defective or n observations (whichever is sooner), the acceptance decision being made if 0 defectives are observed, is seen to require no more observations, for each sample sequence, than any other sampling plan which also gives probability one of acceptance when p (the probability of any item being defective) is 0 and probability of acceptance no greater than that of S when $p=p_1$ (a specified positive number). If $0<p_0<p_1<1$, S can be represented as a SPRT of p_0 against p_1 with bounds satisfying $B<1<A$, provided that $p_1 \leqq n^{-1}$ or that $p_1>n^{-1}$ and p_0 is sufficiently small. Otherwise, S is not a SPRT and there is no SPRT with $B<1<A$ giving the same size and power as S; the SPRT of this form which comes closest and yields at least as much protection requires more observations on the average, and this is the "limitation" of the title. {However, it should be noted that the auth rs do not consider as SPRT's tests for which A and/or B can equal 1 and for which randomization is permitted at the boundaries; it is well known that such tests belong to the minimal complete class (in terms of probabilities of wrong decisions and expected numbers of observations under p_0 and p_1) and possess the Wald-Wolfowitz optimum property. Whenever S is not a SPRT with $B<1<A$, it is actually because S is inadmissible: there is a SPRT with $A=1$ and randomization, and which is better than S (in terms of the same four numbers). Of course, such a SPRT does not satisfy the restriction of acceptance *w.p.*1 when $p=0$, but it still possesses the optimum property with respect to the specified p_0 and p_1.}

The authors also show that Weiss's result [Ann. Math. Statist. **27** (1956), 1178–1181; MR **18**, 957] on the uniqueness of the SPRT with assigned error probabilities can be proved from the more powerful optimum property result. {Reviewer's remark: It is also true that the optimum property can be proved directly from this uniqueness result and the characterization of Bayes procedure.}

J. Kiefer (Ithaca, N.Y.)

11447:

Arrow, Kenneth J. Decision theory and the choice of a level of significance for the t-test. Contributions to probability and statistics, pp. 70–78. Stanford Univ. Press, Stanford, Calif., 1960.

The author reviews and discusses the form of the minimax test concerning the mean of a normal distribution with simple loss function and an indifference zone, and suggests using an approximation of Wallis to compute the test when the variance is unknown.

J. Kiefer (Ithaca, N.Y.)

11448:

Bahadur, R. R. Simultaneous comparison of the optimum and sign tests of a normal mean. Contributions to probability and statistics, pp. 79–88. Stanford Univ. Press, Stanford, Calif., 1960.

For the two tests of the title, let L_0 and L_1 be the (chance) levels at which the sample is "just significant". The limiting joint normal distribution under the null hypothesis, and the strong limit of $(L_0/L_1)^{1/n}$ under any alternative, as the sample size $n\to\infty$, are computed. For a given alternative, this yields a ratio of (large) equivalent sample sizes which, as the alternative approaches the null hypothesis, approaches the Pitman efficiency. More general results in this area appear in the author's paper in Ann. Math. Statist. **31** (1960), 276–295 [MR **22** #7201].

J. Kiefer (Ithaca, N.Y.)

11449:

Bose, R. C. On a method of constructing Steiner's triple systems. Contributions to probability and statistics, pp. 133–141. Stanford Univ. Press, Stanford, Calif., 1960.

The author presents Fisher's and his own solutions for the construction of Steiner systems with 21 and 19

objects, respectively. These were first obtained in 1938, but had never been published previously.

J. Kiefer (Ithaca, N.Y.)

11450:

Bowker, Albert H. A representation of Hotelling's T^2 and Anderson's classification statistic W in terms of simple statistics. Contributions to probability and statistics, pp. 142–149. Stanford Univ. Press, Stanford, Calif., 1960.

P. L. Hsu [Ann. Math. Statist. **9** (1938), 231–243] showed that the noncentral distribution of T^2 is a noncentral F-distribution. The present paper gives a direct representation of T^2 as the ratio of two independent chi-square variables (the numerator being noncentral) and of Anderson's W as a function of two 2×2 Wishart matrices (one noncentral). *J. Kiefer* (Ithaca, N.Y.)

11451:

Bush, Kenneth A. Euler squares. Contributions to probability and statistics, pp. 150–152. Stanford Univ. Press, Stanford, Calif., 1960.

This paper, written before the appearance of the results of Bose, Parker, and Shrikhande, shows that, if $s=2$ (mod 4), then there exist two $s\times s$ Latin squares which are "almost" orthogonal in that at least s^2-s of the s^2 possible ordered pairs arise. *J. Kiefer* (Ithaca, N.Y.)

11452:

Chernoff, Herman. A compromise between bias and variance in the use of non-representative samples. Contributions to probability and statistics, pp. 153–167. Stanford Univ. Press, Stanford, Calif., 1960.

The author shows that if the responses from the various strata of a population vary little in a "neighborhood" of any given stratum, then certain slightly biased estimators of the population characteristic may have smaller mean square error than commonly used unbiased estimators. Examples are included. *J. Kiefer* (Ithaca, N.Y.)

11453:

Connor, W. S. Construction of fractional factorial designs of the mixed 2^m3^n series. Contributions to probability and statistics, pp. 168–181. Stanford Univ. Press, Stanford, Calif., 1960.

Description, with examples, of the construction of Kronecker product designs of the series of the title.

J. Kiefer (Ithaca, N.Y.)

11454:

Dwass, Meyer. Some k-sample rank-order tests. Contributions to probability and statistics, pp. 198–202. Stanford Univ. Press, Stanford, Calif., 1960.

Discussion of various k-sample tests based on combinations of two-sample tests. For previous work on the subject, see Kiefer [Ann. Math. Statist. **26** (1955), 775; **30** (1959), 426–447; MR **21** #1668], Fisz [Bull. Acad. Polon. Sci. Cl. III **5** (1957), 695–698; MR **19**, 693], and H. T. David [Ann. Math. Statist. **29** (1958), 842–851; MR **20** #391].

J. Kiefer (Ithaca, N.Y.)

11455:

Hannan, J. Consistency of maximum likelihood estimation of discrete distributions. Contributions to probability and statistics, pp. 249–257. Stanford Univ. Press, Stanford, Calif., 1960.

From the author's summary: "In dominated classes of discrete distributions, ML estimation can be identified with a certain minimum 'information' estimation and viewed as a particular minimum 'distance' method, permitting a direct extension of the Hotelling consistency proof [Trans. Amer. Math. Soc. **32** (1930), 847–859] under a near minimal assumption, and suggesting examples of inconsistency. Properties of the 'information' convergence topology are studied, e.g., its domination of the L_p topologies. The consistency theorems obtained, but not their proofs, are particular cases of that of Doob [ibid. **36** (1934), 759–775] and that of Kiefer and Wolfowitz [Ann. Math. Statist. **27** (1956), 887–906; MR **19**, 189]."

J. Kiefer (Ithaca, N.Y.)

11456:

Hoeffding, Wassily. An upper bound for the variance of Kendall's "tau" and of related statistics. Contributions to probability and statistics, pp. 258–264. Stanford Univ. Press, Stanford, Calif., 1960.

Let $X_1, \cdots, X_n$ be independently and identically distributed, and let f be bounded, $0\leq f\leq 1$, and satisfy $f(x_1, x_2)=f(x_2, x_1)$. Let $U=2\sum_{i<j}f(X_i, X_j)/n(n-1)$ and $p=Ef(X_1, X_2)$. The author proves that $Ef(X_1, X_2)\times f(X_1, X_3)-p^2\leq h^{3/2}-h^2$, where $h=\max(p, 1-p)$. The resulting bound on var U is illustrated by reference to Kendall's rank correlation coefficient and statistics similar to Wilcoxon's sign-ranked sum and the Cramér-von Mises statistic, conditions for the attainment of the bound being given. *J. Kiefer* (Ithaca, N.Y.)

11457:

Kendall, M. G. The evergreen correlation coefficient. Contributions to probability and statistics, pp. 274–277. Stanford Univ. Press, Stanford, Calif., 1960.

Remarks on various proofs that $E\sin^{-1}r=\sin^{-1}\rho$ (where r and ρ are the sample and normal population correlation coefficients) and of related results.

J. Kiefer (Ithaca, N.Y.)

11458:

Levene, Howard. Robust tests for equality of variances. Contributions to probability and statistics, pp. 278–292. Stanford Univ. Press, Stanford, Calif., 1960.

Let X_{ij} $(1\leq i\leq p,\ 1\leq j\leq N_i)$ be independent and, for each i, identically distributed with mean μ_i and variance σ_i^2. Let $\bar{X}_i=\sum_j X_{ij}/N_i$ and $Z_{ij}=|X_{ij}-\bar{X}_i|$. To test the equality of the values σ_i^2, the author suggests computing a one-way F statistic based on the random variables z_{ij} or z_{ij}^2 or $\log z_{ij}$ or $z_{ij}^{1/2}$. The first of these possibilities seemed to yield the best overall "robustness" results in sampling experiments with normal and certain non-normal X_{ij}. The results are compared with those of Bartlett, Box, and Andersen. *J. Kiefer* (Ithaca, N.Y.)

11459:

Mann, Henry B.; Menon, M. V. Intrablock and interblock estimates. Contributions to probability and statistics, pp. 293–298. Stanford Univ. Press, Stanford, Calif., 1960.

Discussion of incomplete block designs and partially confounded designs whose analysis can be reduced to one

wherein there are independent normal variables X_i and Y_j with variances σ_1^2 and σ_2^2 and means c_iS_i and d_jS_j, the S_i being 0 for $i>p$, and the other S_i and σ_j being unknown. Maximum likelihood estimation of S_i involves the solution of a cubic. The authors show that a customarily used estimate differs from the ML estimate by a smaller order in probability than the standard deviation of the latter, as the number of X_i and Y_i with 0 mean becomes large. *J. Kiefer* (Ithaca, N.Y.)

11460:

Nicholson, George E., Jr. Prediction in future samples. Contributions to probability and statistics, pp. 322–330. Stanford Univ. Press, Stanford, Calif., 1960.

This paper studies the degree to which the multiple correlation coefficient of a first sample (in a standard normal model) tends to exceed the multiple correlation coefficient between the observed criterion in a second sample and those estimated from the predictors of the second sample using regression coefficients calculated from the first. *J. Kiefer* (Ithaca, N.Y.)

11461:

Pendergrass, R. N.; Bradley, R. A. Ranking in triple comparisons. Contributions to probability and statistics, pp. 331–351. Stanford Univ. Press, Stanford, Calif., 1960.

From the authors' summary: "A mathematical model for triple comparisons has been derived as an extension of the model for paired comparisons. Test procedures are derived. The main results are asymptotic ones. Asymptotic relative efficiencies of triple comparisons relative to paired comparisons and the analysis of variance are respectively 3/2 and $9/4\pi$." *J. Kiefer* (Ithaca, N.Y.)

11462:

Robbins, Herbert. A statistical screening problem. Contributions to probability and statistics, pp. 352–357. Stanford Univ. Press, Stanford, Calif., 1960.

Let X be normal with mean θ and known variance c^2, where θ has a priori normal distribution with known mean μ and variance σ^2. The gain is θ (positive or negative) if decision d_1 is made, 0 if d_2 is made. The Bayes rule is tabled and discussed. The classical Bayes estimator for squared error loss function is also discussed.
J. Kiefer (Ithaca, N.Y.)

11463:

Rosenblatt, Joan Raup. On the power of some rank-order two-sample tests. Contributions to probability and statistics, pp. 358–370. Stanford Univ. Press, Stanford, Calif., 1960.

Let $X_1, \cdots, X_n$ with common (univariate) d.f. F and $Y_1, \cdots, Y_m$ with common d.f. G be independent, and let $\theta(F, G)=P\{X_1<Y_1\}$. Let $\phi(x, y)=1$ or 0 according to whether or not $x>y$, and let $mn\,U_{mn}=\sum_{i,j}\phi(x_i, y_j)$ and, assuming $n\leq m$, $nW_{mn}=\sum_i\phi(x_i, y_i)$. Using the binomial character of nW_{mn} and certain "least favorable" F and G, it is shown that a one- [respectively, two-] tail test based on W_{mn} maximizes the minimum power for testing $\theta\leq\theta_1$ against $\theta\geq\theta_2$ where $\theta_1<\theta_2$ [respectively, $\theta=\frac{1}{2}$ against $|\theta-\frac{1}{2}|\geq\delta>0$] among (F, G) with densities with respect to a fixed measure. For size less than θ_1^s where $s=\min(m, n)>1$ [respectively, less than $2^{-(s-1)}$], the corresponding Wilcoxon-Mann-Whitney tests based on U_{mn} are inadmissible for (F, G) continuous. The results for testing $F=G$ against one of the same alternatives with a rank order test depend on α and θ_2 (or δ), and, as shown in a table (for $1\leq m\leq 2\leq n\leq 3$), can be in the same or opposite direction from those just cited for testing $\theta\leq\theta_1$ (or $\theta=\frac{1}{2}$). *J. Kiefer* (Ithaca, N.Y.)

11464:

Roy, S. N.; Bhapkar, V. P. Some non-parametric analogs of "normal" ANOVA, MANOVA, and of studies in "normal" association. Contribution to probability and statistics, pp. 371–387. Stanford Univ. Press, Stanford, Calif., 1960.

The model is that of a product of several multinomial distributions yielding frequencies in a multiway classification. Large sample tests are suggested for various hypotheses. The paper is a sequel to a sequence of papers by Roy and others [see, e.g., S. N. Roy, *Some aspects of multivariate analysis*, Wiley, New York, 1957; MR **19**, 1093]. *J. Kiefer* (Ithaca, N.Y.)

11465:

Shrikhande, S. S. Relations between certain incomplete block designs. Contributions to probability and statistics, pp. 388–395. Stanford Univ. Press, Stanford, Calif., 1960.

The author establishes relations, too numerous and lengthy to quote, between various balanced incomplete block designs derived from symmetrical balanced incomplete block designs with $\lambda=1$ and 2, and certain partially balanced incomplete block designs.
J. Kiefer (Ithaca, N.Y.)

11466:

Solomon, Herbert. Classification procedures based on dichotomous response vectors. Contributions to probability and statistics, pp. 414–423. Stanford Univ. Press, Stanford, Calif., 1960.

The procedures considered for the problem of the title are the likelihood ratio procedure and several other more easily computable procedures, each individual's observed n-vector of 0's and 1's having one of two distributions. An example is considered, and the risk functions for the procedures are computed for $n\leq 6$.
J. Kiefer (Ithaca, N.Y.)

11467:

Stein, Charles. Multiple regression. Contributions to probability and statistics, pp. 424–443. Stanford Univ. Press, Stanford, Calif., 1960.

Let $X_1, \cdots, X_n$ be independent, normal $(p+1)$-vectors with mean ξ and covariance matrix Σ. The object is to estimate the regression coefficients for the regression (including the constant term) of the first component of X_i on the last p components Z_i of X_i, with an appropriate quadratic loss function. First the author proves the minimax character of the ML estimator when all parameters are unknown, or when ξ is known, or when the distribution of Z_i and the conditional variance of the first component are known, by applying a theorem of the reviewer [Ann. Math. Statist. **28** (1957), 573–601; MR **19**, 1097]; it is crucial that a small enough group yields the

best invariant estimator, since the full group which leaves the problem invariant is too large for the minimax theorem to hold. Next, using tools developed previously by the author [Proc. Third Berkeley Symposium on Mathematical Statistics and Probability, 1954–1955, vol. I, pp. 197–206, Univ. of California Press, Berkeley, Calif., 1956; Ann. Math. Statist. **30** (1959), 970–979; MR **18**, 948; **22** #278], he proves that the estimator is admissible when ξ is unknown, $p=1$ and $n\geqq 6$; and that when ξ is known the estimator is admissible if $p=1$, $n\geqq 4$ or $p=2$, $n\geqq 6$, and inadmissible if $p\geqq 3$ (or trivially, if $n\leqq p+1$). Improvements on the ML estimator in the cases when it is inadmissible are indicated. *J. Kiefer* (Ithaca, N.Y.)

11468:

Terry, Milton E. An optimum replicated two-sample test using ranks. Contributions to probability and statistics, pp. 444–447. Stanford Univ. Press, Stanford, Calif., 1960.

A rank order test of the hypothesis that k pairs of random samples, the ith pair containing m_i and n_i observations, are all independent and are distributed with all observations in the ith pair having the same d.f. (depending on i), and which is most powerful among rank order tests against certain local normal alternatives, is characterized. Tables for small sample sizes, and asymptotic (normal) distributions, are given.

J. Kiefer (Ithaca, N.Y.)

11469:

Tukey, John W. A survey of sampling from contaminated distributions. Contributions to probability and statistics, pp. 448–485. Stanford Univ. Press, Stanford, Calif., 1960.

A study centering around the question of relative asymptotic efficiency of the sample standard deviation and mean deviation for estimating the population standard deviation of a mixture of two normal distributions with equal means. *J. Kiefer* (Ithaca, N.Y.)

11470:

Wilks, S. S. Multidimensional statistical scatter. Contributions to probability and statistics, pp. 486–503. Stanford Univ. Press, Stanford, Calif., 1960.

A survey of properties of the generalized variance, Hotelling's T^2, principal components, discriminant analysis, and canonical correlation, all treated from the point of view of "statistical scatter".

J. Kiefer (Ithaca, N.Y.)

11471:

Zelen, M. Analysis of two-factor classifications with respect to life tests. Contribution to probability and statistics, pp. 508–517. Stanford Univ. Press, Stanford, Calif., 1960.

From the author's summary: "This paper outlines [maximum likelihood and likelihood ratio] procedures for analyzing a two-way classification with respect to the scale factor of the exponential distribution. Approximations are given for tests based on small samples."

J. Kiefer (Ithaca, N.Y.)

11472:

Solomon, Herbert. Classification procedures based on dichotomous response vectors. Studies in item analysis and prediction, pp. 177–186. Stanford Univ. Press, Stanford, Calif., 1961.

Reprinting of article reviewed in #11466.

11473:

Šalaevskiĭ, O. V. Some remarks on the adjustment of observations with unknown weights. Dokl. Akad. Nauk SSSR **130** (1960), 37–40 (Russian); translated as Soviet Math. Dokl. **1**, 27–30.

This is an extension of Wald's approach to the Behrens-Fisher problem [*Selected papers in statistics and probability by Abraham Wald*, McGraw-Hill, New York, 1955, 669–695; MR **17**, 55], combined with a method of constructing confidence ellipsoids considered by Linnik [Teor. Veroyatnost. i Primenen. **2** (1957), 349–359; MR **19**, 1096] in the case of known variance ratios. There are r groups of mutually independent, normally distributed random variables (which are observed) whose means are given linear functions of unknown parameters $\xi_1, \cdots, \xi_m$ and whose variances are equal within groups and are unknown. The object is to estimate a vector-valued linear function H of $\xi_1, \cdots, \xi_m$ with $k\leqq m$ components. The matrices involved are assumed to have maximal rank. Let $\bar{H}$ be the least-squares estimator of H when the variances $\sigma_1^2, \cdots, \sigma_r^2$ are all equal, and M_0 a certain $k\times k$ matrix which depends on estimators s_a^2 of σ_a^2. Theorem 1 states that for any positive integer q there exists a function $V_q=V_q(c, s_1^2, \cdots, s_r^2)$ $(c\geqq 0)$ such that

$$P\{(H-\bar{H})'M_0^{-1}(H-\bar{H}) \leqq V_q\} = P\{\chi_k^2 \leqq c\}+R_q,$$

where χ_k^2 has the chi-square distribution with k degrees of freedom, $|R_q|<Cn^{-q}$, and C depends on r and m only. Theorem 4 gives an analogous result (for a special case) in terms of Fisher's F instead of χ^2. The proofs are only briefly sketched. *W. Hoeffding* (Chapel Hill, N.C.)

11474:

Khamis, Salem H. Incomplete gamma functions expansions of statistical distribution functions. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 385–396. (French summary)

The author considers the expansions of a cumulative distribution function $F(x)$, $0<x\leqq\infty$, and of the corresponding probability density $f(x)=F'(x)$ into series involving orthogonal Laguerre polynomials $L_r(x)=L_r(n, \alpha, x)$, which he defines by the relation $\theta_n(x)L_r(x)=(-d/dx)^r\{x^r\theta_n(x)\}$ $(r=0, 1, \cdots)$, where $\theta_n(x)=\alpha^n x^{n-1}e^{-\alpha x}/\Gamma(n)$, and n and α are positive real numbers. He develops formulae designed to facilitate the computation of the coefficients and terms of these expansions. He writes μ_t, ν_t for the moments of order t belonging to $f(x)$, $\theta_n(x)$, respectively, and puts $M_t=\mu_t/\nu_t$, $A_r=(\nu_r/r!)[\Delta^r M_t]_{t=0}$, where Δ^r indicates an advancing difference of the rth order. Then $L_r(x)\theta_n(x)=\nu_r\Delta^r\theta_n(x)$ and, formally, $f(x)=\sum_{r=0}^{\infty}A_r\Delta^r\theta_n(x)$, and there is a similar expansion for $F(x)$. These expansions can easily be transformed into series in the functions $\theta_{n+k}(x)$ $(k=0, 1, \cdots)$, the integrals of which are incomplete gamma-functions. (When $2n$ is an integer they correspond to χ^2-distributions.) The author gives a numerical example

illustrating the computations involved. He also discusses the inversion of cumulative distribution functions of the type considered by means of the Newton-Raphson iteration or a generalization of it.

H. O. Mulholland (Exeter)

11475:

Lehman, Shirley Young. Exact and approximate distributions for the Wilcoxon statistic with ties. J. Amer. Statist. Assoc. **56** (1961), 293–298.

The exact distribution of the Wilcoxon rank-sum statistic is calculated for five specific cases, each case consisting of two independent samples of size five, including some ties. Ties are treated by the mid-rank method. The normal approximation of Kruskal and Wallis, particularly with continuity correction, is usually adequate, except when the number of ties is excessive.

E. S. Keeping (Edmonton, Alta.)

11476:

Scheuer, Ernest M. The cumulative distribution of the sum or difference of a normal random variable and the absolute value of a normal random variable. SIAM Rev. **3** (1961), 152–155.

Let x and y be normally distributed random variables, with means μ_x and μ_y, variances σ_x^2 and σ_y^2, and correlation ρ. The author finds the cumulative distribution of $x+|y|$ and $x-|y|$. *J. L. Snell* (Hanover, N.H.)

11477:

Curnow, R. N. The consequence of errors of measurement for selection from certain non-normal distributions. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 291–308. (French summary)

The author studies the conditional distribution (especially the moments) of x, given that $y=x+w>\eta$, where w is independent of x and normal, and η is a specified fractile of the distribution of y. (x is a characteristic of interest, w is error, and values of $y>\eta$ are "selected".) Results are obtained in terms of tabulated functions when x has (unconditionally) a normal, rectangular, gamma, χ, reversed gamma, or reversed χ distribution.

J. W. Pratt (Cambridge, Mass.)

11478:

Nadler, Jack. Inverse binomial sampling plans when an expontential distribution is sampled with censoring. Ann. Math. Statist. **31** (1960), 1201–1204.

Let X be a random variable with c.d.f. $F(x)=1-e^{-x/\theta}$ for $x>0$, and zero otherwise. Observations from this distribution are censored at $x=x_0$. The following sample plan is proposed: Sample until r uncensored observations are obtained. Denote by $X_1, \cdots, X_r$ these r uncensored observations and by N the total (random) number of observations. Further, let $S=\sum_{i=1}^{r} X_i+(N-r)x_0$. Then it is shown that with this sampling plan S is a complete sufficient statistic for θ and $2S/\theta$ has a χ^2 distribution with $2r$ degrees of freedom. *D. G. Chapman* (Seattle, Wash.)

11479:

Nagar, A. L. The bias and moment matrix of the general k-class estimators of the parameters in simultaneous equations. Econometrica **27** (1959), 575–595.

The author considers the k-class estimators introduced by Theil [*Economic forecasts and policy*, North-Holland, Amsterdam, 1958; Ch. VI], where $k-1$ is of the order of T^{-1}, where T is the sample size. He obtains the asymptotic bias, to terms of order T^{-1}, and the asymptotic variance, to terms of order T^{-2}, of the estimates, and from these obtains an "unbiased" and a "minimum generalized second moment" estimator.

H. Rubin (E. Lansing, Mich.)

11480:

Nagar, A. L. A Monte Carlo study of alternative simultaneous equation estimators. Econometrica **28** (1960), 573–590.

The author compares classical least-squares estimators with three of the k-class estimators of the preceding review: two-stage least squares, "unbiased", and "minimum second moment", by Monte Carlo methods. {In the reviewer's opinion, the sample sizes are too small to indicate clearly the nature of the results, even if the numbers obtained would (which they do not).} The biases and second moments in Table VIII are the asymptotic expressions in the author's previous paper [#11479] and are not the true values. *H. Rubin* (E. Lansing, Mich.)

11481:

Swamy, P. S. Estimating the mean and variance of a normal distribution from singly and doubly truncated samples of grouped observations. Calcutta Statist. Assoc. Bull. **9** (1960), 145–156.

Author's summary: "The maximum likelihood estimating equations are derived to estimate the mean and variance of a normal distribution from singly and doubly truncated samples of grouped observations with known truncation points, when the number of unmeasured observations is (1) unknown for each 'truncated tail', (2) known separately for each 'truncated tail', (3) known jointly for the two 'truncated tails'. The equations are seen to be easily solvable with the aid of Gjeddebaek's tables [Skand. Aktuarietidskr. **32** (1949), 135–159; MR **11**, 446] of Z_1 and Z_2 functions. Asymptotic variance-covariance matrix of the estimates is obtained in each of the three cases, and an investigation is made into the loss of information due to truncation of the samples. The corresponding results for the singly truncated samples are obtained as special cases of (1) and (2). Practical application of the results has been illustrated by a numerical example." *J. Janko* (Prague)

11482:

Leone, F. C.; Rutenberg, Y. H.; Topp, C. W. The use of sample quasi-ranges in setting confidence intervals for the population standard deviation. J. Amer. Statist. Assoc. **56** (1961), 260–272.

Authors' summary: "The problem is the determination of an optimal selection method of quasi-range for setting one-sided confidence bounds and confidence intervals for the standard deviation from a given distribution. The proposed methods of optimal selection are applied to random ordered samples from the normal, exponential and rectangular distributions. Tables of confidence bounds for the standard deviation of these distributions are given for confidence levels commonly used in statistical work. These are compared with the results of standard procedures.

"For sample sizes between 10 and 100, our intervals are roughly 3 to 2 times longer than the ones obtained by the conventional approach. Our samples have to be roughly 3 to 2 times larger to yield a confidence interval of a given expected size. The principal advantage of our method is in terms of computation time. With the aid of the tables, all that is required is the ordering of a few lowest and a few highest sample values and calculating two differences and two products." *D. Teichroew* (Stanford, Calif.)

11483:

DeGroot, M. H. Minimax sequential tests of some composite hypotheses. Ann. Math. Statist. **31** (1960), 1193–1200.

Let $X(t)$, $t \geqq 0$ be a Wiener process with unknown mean μ per unit time and unit variance per unit time. Further $\Pr[X(0)=0]=1$. The process may be observed continuously for $t \geqq 0$ and we consider a test of the hypothesis $H_0: \mu \leqq 0$ against the alternative $H_1: \mu > 0$ based on such a sample realization of the process. If the cost of observing the process is a linear function of t and the cost of a wrong decision is of the form $c|\mu|^r$, $c>0$, $0<r\leqq 2$ then a certain sequential probability ratio test (SPRT) is minimax decision procedure. The method of proof is to find the minimax symmetric SPRT and then to show that this is minimax in the class of all decision procedures. Use is also made of several results of Dvoretzky, Kiefer and Wolfowitz [same Ann. **24** (1953), 254–264; MR **14**, 997].

D. G. Chapman (Seattle, Wash.)

11484:

Hemelrijk, Jan. Distribution-free tests against trend and maximum likelihood estimates of ordered parameters. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 15–25. (French summary)

From the author's summary: ". . . a review is given of a number of publications on the two subjects mentioned in the title. These subjects have been treated separately in the literature so far and the methods of treatment also are different. On the other hand the problems are closely related. . . ." *D. L. Burkholder* (Urbana, Ill.)

11485:

Gates, Charles E.; Beard, Benjamin H. Rapid chi-square test of significance for three-part ratios. Biometrics **17** (1961), 261–266.

A quick graphical method is developed for testing by chi-square the goodness-of-fit of the observed frequencies in a large sample divided into three classes, the assumed population ratio for the three classes being given by genetic or other theory. The significance regions are bordered by ellipses. A table is given for constructing the ellipses for given sample size and for some common genetic ratios (such as 1:2:1).

E. S. Keeping (Edmonton, Alta.)

11486:

Pompilj, Guiseppe. On entirely pseudo-linear regression. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 60–63. (French summary)

The author presents characteristics of random double variables (x, y) ". . . with entirely pseudo-linear regression on the hypothesis that there exist and are finite all the moments of a finite order, and that the random variable itself has a non-null superficial variability". The results are given in the form of theorems. No proofs are included in this paper but reference to such proofs is given. The author also makes frequent reference to his earlier work in this area.

In introducing the concept of entirely pseudo-linear regression, the author refers to the use of polynomials as approximations to the regression function. He explains, "If the regression is linear, all the least squared polynomials are linear but . . . this proposition is not invertible and therefore when all the least squared polynomials are linear the regression will be pseudo-linear." Double variables with entirely linear regression are those which have a linear regression whatever direction is considered. Random double variables with entirely pseudo-linear regression are those which present a pseudo-linear regression in all directions.

P. S. Dwyer (Ann Arbor, Mich.)

11487:

Greenberg, Bernard G.; Sarhan, Ahmed E. Some applications of order statistics. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 172–183. (French summary)

This is an expository paper. The uses of recent contributions to the techniques in order statistics have been applied in four instances. The first is in the case of censored observations from the normal distribution. The second application involves estimation of the parameters of an exponential distribution using $k<n$ of the sample elements. The third application is in the grouping of continuous data into frequency classifications. The final illustration is an application of the use of order statistics in the short-cut analysis of variance.

D. G. Chapman (Seattle, Wash.)

11488:

Capon, Jack. Asymptotic efficiency of certain locally most powerful rank tests. Ann. Math. Statist. **32** (1961), 88–100.

Let $X_1, \cdots, X_m$, $Y_1, \cdots, Y_n$ be mutually independent random variables, where each X_i (Y_j) has the continuous distribution function $F_X(F_Y)$. For each τ in a non-degenerate interval let G_τ be a continuous distribution function. Under certain regularity conditions the rank test for testing $F_X=F_Y$ which is most powerful against the alternative $F_X=G_\theta$, $F_Y=G_\phi$ is derived, where θ and ϕ are specified numbers sufficiently close to a number ϕ_0. [See also H. Uzawa, same Ann. **31** (1960), 685–702; MR **22** #8608.] It is shown that this rank test has asymptotic efficiency one in Pitman's sense. The proof depends on a theorem of H. Chernoff and I. R. Savage [ibid. **29** (1958), 972–994; MR **20** #6755], who also obtained the present result for the case of translation alternatives. *W. Hoeffding* (Chapel Hill, N.C.)

11489:

Graybill, Franklin A.; Hultquist, Robert A. Theorems concerning Eisenhart's model II. Ann. Math. Statist. **32** (1961), 261–269.

Given the general variance component model

$$Y_m = \mu X_0 + \sum_{i=1}^{k} \sum_{j=1}^{p_i} \beta_{ij} X_{ijm} + e_m,$$

where $m=1, 2, \cdots, n$; $\{\beta_{ij}\}$ are independent random variables with variances σ_i^2 and zero means and similarly for the e_m, with variances σ_{k+1}^2; the X_{ijm} are known constants ($X_0 \equiv 1$). Let X_i' be the $(p_i \times n)$ matrix (X_{ijm}). It is assumed that the $(n \times n)$ matrices $A_i = (X_i X_i')$, including A_0 (a matrix of ones) and A_{k+1} (the unit matrix) commute one with the other. Using this assumption plus $\sum_{m=1}^{n} X_{ijm} = r_i$ and $\sum_{j=1}^{p_i} X_{ijm} = 1$, it is shown that the number of distinct characteristic roots (s) of W, the matrix of the expected value of $(\mathbf{YY}')$, is not less than $k+2$, where $\mathbf{Y}$ is the vector of Y's; also that the maximum number of distinct roots of the covariance matrix of $\mathbf{Y}$ is one plus the rank of the matrix of X's, including $\mathbf{X}_0$.

Assuming that the A_i commute and that the β's and e's have multinormal densities, the authors derive a set of s minimal sufficient statistics for the variance component model, based on the s distinct characteristic roots of W. If $s=k+2$, this set is complete. An example is given of a half-replicate of a 4×4 factorial-type model with no interaction component (four parameters) for which the minimal set consists of five statistics. It is not indicated how these five statistics should be used to estimate the four parameters.

Each variance component is estimable, that is, the expected value of some quadratic form of the Y's equals the component, if and only if the A_i are linearly independent. Under the normality assumption, an analysis of variance is said to exist if $\sum_m Y_m^2$ is the sum of $k+2$ pairwise independent quadratic forms, each distributed as a noncentral χ^2-variate divided by a different linear function of the parameters. This exists if and only if the A_i commute and W has $k+2$ distinct characteristic roots.

{In this reviewer's opinion, the restrictive assumptions needed for the theorems in this and similar articles severely limit their usefulness as guides in designing experiments to obtain good estimates of all the variance components.} *R. L. Anderson* (Raleigh, N.C.)

11490:

Kitagawa, Tosio. Successive process of statistical inferences applied to linear regression analysis and its specialisations to response surface analysis. Bull. Math. Statist. **8**, 80–114 (1959).

Consider the linear regression model

$$y_\alpha = \varphi_q(x_{1\alpha}, x_{2\alpha}, \cdots, x_{q\alpha}) + \varepsilon_\alpha \quad \text{for} \quad \alpha = 1, 2, \cdots, N,$$

where $\varphi_q(x_{1\alpha}, x_{2\alpha}, \cdots, x_{q\alpha}) = \sum_{i=1}^{q} \beta_i x_{i\alpha}$ and the $\{\varepsilon_\alpha\}$ are independent $N(0, \sigma^2)$. A discussion is given of estimation procedures termed "two-stage sequential" and "three-stage sequential". A two-stage sequential procedure is when the first stage consists of taking N_1 observations on y_α and making a test of significance on the null hypothesis H_0: $\beta_{p+1} = \beta_{p+2} = \cdots = \beta_q$. If the test is not significant, the experiment is stopped and the estimating procedure is to take as the model $E(y_\alpha) = \varphi_p(x_{1\alpha}, x_{2\alpha}, \cdots, x_{p\alpha})$ and solve the resulting normal equations for β_i ($i = 1, 2, \cdots, p$). When the test is significant, take additional observations and use the data in both stages to estimate the unknown parameters in the model $E(y_\alpha) = \varphi_q(x_{1\alpha}, x_{2\alpha}, \cdots, x_{q\alpha})$. The mean square error of the predicting equation is derived for the procedure. Similar results are given for the three-stage sequential procedure. Applications are made to rotatable designs of order two and three in two variables.

M. Zelen (Washington, D.C.)

11491:

Shirafuji, Michie. A two stage sequential design in response surface analysis. Bull. Math. Statist. **8**, 115–126 (1959).

Consider a k-dimensional response surface of order two having the model

$$y_\alpha = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \varepsilon_\alpha,$$

where $\{\varepsilon_\alpha\}$ are independent $N(0, \sigma^2)$. The author discusses a two-stage procedure for estimating $E(y_\alpha)$. The first stage consists of a 2^k factorial design. A test of significance is made comparing the pooled sum of squares associated with all interaction terms (β_{ij}, $i \neq j$) against the residual sum of squares. If the test is significant, experimentation is stopped and one regards the model as having the β_{ii} terms absent. When the test is not significant, $(2k+1)$ additional experimental points are taken so that the pattern of experimentation correspond to a composite design. The composite design is then used to estimate all $\binom{k+2}{2}$ parameters. The main result is the calculation of the norm

$$E\left\{ \int_{\mathscr{D}_R} \cdots \int |\hat{y}(x_1, x_2, \cdots, x_k) - E[\hat{y}(x_1, x_2, \cdots, x_k)]|^2 \, dx_1 dx_2 \cdots dx_k \right\}$$

where the domain of integration is taken to be $\mathscr{D}_R : \sum_{i=1}^{k} x_i^2 \leq R$ and $y(x_1, x_2, \cdots, x_k)$ is the predicted value of the response at the point $(x_1, x_2, \cdots, x_k)$. Tables are given for evaluating the norm when the β_{ij} and β_{ii} terms are taken to be equal (not necessarily to each other). *M. Zelen* (Washington, D.C.)

11492:

Rao, P. V. The dual of a balanced incomplete block design. Ann. Math. Statist. **31** (1960), 779–785.

The author has considered the general nature of a design known as the 'dual' of a b.i.b. design, and investigated the conditions under which it satisfies the definition of a partially balanced incomplete block design without restriction of the number of associate classes to two only. He has utilised the properties of the 'incidence matrix', N, its transpose, N', and the products NN', $N'N$ and $N'NN'N$ to deduce further conditions over and above those mentioned by W. S. Connor and Q. M. Husain in connection with the parent b.i.b. designs. This has resulted in a general theorem which includes, as particular cases, the theorems of Shrikhande for $\lambda=1$ and $\lambda=2$. The usefulness of the theorem has been illustrated in finding the P-matrices of the 3-associates case of the truncated design obtained from the symmetrical design $v=b=25$, $r=k=9$, $\lambda=3$.

The exposition is clear, though the section 4 explaining 'the dual of a design' could have been expressed in a simpler way than in a mathematically formal way without sacrificing rigour. *Q. M. Husain* (Dacca)

11493:

Finney, D. J. ★An introduction to the theory of experimental design. The University of Chicago Press, Chicago, Ill., 1960. xii + 223 pp. $7.00.

The title indicates the contents. The book is intended for mathematicians with some knowledge of statistics, but contains little formal manipulation. Instead, the author tries to make the general structure of the theory clear, leaving the reader to fill in details, or find them elsewhere. Throughout the book attention is drawn to the relation between theoretical specification and practical situation. The subjects covered are as follows: (Chapter 1) general practical considerations involved in designing experiments requiring statistical analysis; (Chapters 2 to 7) the analysis of variance, and an outline of standard designs (factorial, confounded, with fractional replication, split plots, incomplete blocks, cross-over designs, etc.); (Chapter 8) sequential experimentation, and the estimation of optimal conditions (as in chemical experiments); (Chapter 9) efficiency of experimentation, i.e., the choice of designs which accomplish the desired ends at minimum cost (labor, etc.), including some selection procedures, applicable to crop varieties, drugs, etc.; (Chapter 10) 'external' economics of experimentation, i.e., how many experiments are needed to give maximum economic benefit to the community. *C. A. B. Smith* (London)

11494:

Cox, D. R. Design of experiments: the control of error. J. Roy. Statist. Soc. Ser. A **124** (1961), 44–48.

A "review in general terms of the principles of [experimental] design that are available for the control of error in comparative experiments."

H. L. Seal (New Haven, Conn.)

11495:

Kudô, Akio. The classificatory problem viewed as a two-decision problem. Mem. Fac. Sci. Kyushu Univ. Ser. A **13** (1959), 96–125.

Samples have been drawn from K populations with distribution functions $F(y: \theta_i)$, $\theta_i \in \Omega$, $i=1, 2, \cdots, K$, where Ω is given but except for this inclusion, the θ_i are unknown. The problem is to classify an observation y (assumed to originate from one of these K populations) as to its population of origin. A vector-valued classification function (which determines a classification procedure and is in a sense analogous to a randomized test) and its power and generalized power are defined. It is shown that in terms of power, immediate restriction may be made to those classification functions which depend on the K samples only through statistics sufficient for the family of joint distributions with $\theta_i \in \Omega$, $i=1, \cdots, K$. Invariance of classification functions under groups of transformations (change of scale, change of scale and location) is defined. For $K=2$, four cases are considered: univariate (multivariate) normal populations with unknown means (mean vectors) and known and unknown common variance (variance, covariance matrix). The standard classification procedure for discriminating between two normal populations of unknown means and common known (unknown) variance, generalized to allow for unequal sample sizes, is shown to be optimum powerwise within a class of location (location and scale) invariant classification functions with power functions that satisfy certain symmetry conditions. Sufficient conditions are given for the above types of invariance. Generalizations are made to the multivariate case, when the variance covariance matrix is known. Restriction of consideration to classification functions invariant under the above transformation groups is justified on an intuitive basis.

M. Skibinsky (W. Lafayette, Ind.)

11496:

Kudô, Akio. The classificatory problem viewed as a two-decision problem. II. Mem. Fac. Sci. Kyushu Univ. Ser. A **14** (1960), 63–83.

This paper is an extension of part I [#11495] under the same title. The envelope power of a collection of classification functions, stringency within the class, and most stringent classification function are defined. The notion of symmetry of a classification function relative to distance functions on the parameter space is used to define a restricted collection of classification functions (serving a purpose somewhat analogous to randomized tests of prescribed significance level) within which a most stringent classification function may be sought. It is shown that the standard classification procedure for discriminating between two normal populations of unknown means and common known (or unknown) variance is most stringent among all classification procedures symmetric relative to natural distance functions on the parameter space. This result is generalized to the multivariate case. Modifications of theorems due to Riesz, and Hunt and Stein, required for demonstration of the above results, are proved at the outset. *M. Skibinsky* (W. Lafayette, Ind.)

11497:

Gupta, Shanti S.; Sobel, Milton. Selecting a subset containing the best of several binomial populations. Contributions to probability and statistics, pp. 224–248. Stanford Univ. Press, Stanford, Calif., 1960.

A procedure is suggested for selecting a subset of k binomial populations which, with specified probability γ, contains that population with the largest "p" value. The approximate expected size of the subset is tabled as a function of sample size, γ, and the difference between the largest and other (assumed equal) values of p. A discussion of several other procedures is given.

J. Kiefer (Ithaca, N.Y.)

11498:

Weber, Erna. Das Ergebnis-Folge-Verfahren (Sequenzanalyse). Grundlagen und Anwendungen. (1. Fortsetzung). Wiss. Z. Humbolt-Univ. Berlin Math.-Nat. Reihe **8** (1958/59), 519–534. (Russian, English, and French summaries)

Expository paper on the sequential test for the binomial distribution with medical applications and worked examples. *D. G. Chapman* (Seattle, Wash.)

11499:

Hoeffding, Wassily. Lower bounds for the expected sample size and the average risk of a sequential procedure. Ann. Math. Statist. **31** (1960), 352–368.

Let $X_1, X_2, \cdots$ be a sequence of independent identically distributed random variables having a common probability density f with respect to μ. Let f_1, f_2 be two p.d. such that decision d_2 [d_1] is wrong if $f=f_1$ [f_2]. The sequential tests, for making decision d_i, considered are those for which the probability of a wrong decision does not exceed α_i (>0) when $f=f_i$. Let N denote the number of observations

required by the test. The author presents a detailed discussion of lower bounds for $E_0(N)$, i.e., when $f=f_0$, not necessarily the same as f_1 or f_2. Two new inequalities are obtained. Define $\zeta=\max(\zeta_1, \zeta_2)$, $\zeta_i=E_0 \log (f_0/f_i)$, $\tau^2=E_0[\log (f_2/f_1)-\zeta_1+\zeta_2]^2$; then

$$(1) \qquad E_0(N) \geq (1-\alpha_1-\alpha_2)[1-\int \min(f_0, f_1, f_2)\, d\mu]^{-1},$$

and with some additional assumptions,

$$(2) \qquad E_0(N) \geq \{[(\tau/4)^2-\zeta \log (\alpha_1+\alpha_2)]^{1/2}-\tau/4\}^2\zeta^{-2}.$$

The attainment of equality in (1) and near attainment in (2) is discussed. In particular, the test of Anderson [same Ann. **31** (1960), 165–197; MR **22** #7229] yields results close to (2) for the examples considered. A lower bound for the average risk is also obtained, and this leads to a sequence of lower bounds for $E_0(N)$, which are in general better than (1). *I. Olkin* (Minneapolis, Minn.)

11500:

Sibuya, Masaaki. Cutting out procedures for material with Poisson defects. Ann. Inst. Statist. Math. **12** (1960), 151–159.

Consider one-dimensional material such as wire with defects λ per unit length, a Poisson process. The material is cut into strips. If a strip has c or fewer defects, it is accepted, otherwise it is rejected. Four procedures are given for cutting the strips and the yield is determined in each case, either theoretically or by Monte Carlo techniques. *L. A. Aroian* (Los Angeles, Calif.)

11501:

Romig, Harry G. Recent advances in sampling techniques. Estadística **17** (1959), 778–797. (Spanish summary)

This is an expository paper concerning current methods of sampling inspection in quality control.

C. Hayashi (Tokyo)

11502:

Deming, W. Edwards. ★Sample design in business research. A Wiley Publication in Applied Statistics. John Wiley & Sons, Inc., New York-London, 1960. xx+517 pp. $12.00.

This book on the art and its supporting theory of sampling in business will go a long way toward explaining the role of the consulting statistician to his business clients as well as to the student of sampling. The major problem treated is the design of the sampling procedure for estimating indices of the economic state of a large firm or of a sector of the economy, and interpreting the sampling and non-sampling errors in the results.

Such indices are frequently used as legal evidence, and consequently the author has provided (in Part I) a code of professional ethics to guide the sampling statistician's intervention in these surveys. If some of the pronouncements on "outside margins of error" (page 13) and "This statement is not a matter of opinion: it is a mathematical consequence of the sampling procedure . . ." (page 18) appear overly doctrinaire, they are tempered by others such as "The statistician should not, in my judgment, give advice to the client to tell him what decision he ought to make" (page 13).

Non-sampling errors are distinguished from sampling errors by explicit use of the concept "equal complete coverage", and implicit use of the concept "ideal complete coverage". Dependence on random-number tables in sample selection gives rise to variations in the estimate about a value which would have been obtained from a complete-coverage survey run in the same way as the sample survey, while the shortcomings of the equal complete-coverage value are discovered by applying the author's recommended auditing procedures.

The use of replication in the form of independently drawn subsamples becomes incorporated (in Part II) into other devices such as stratification and the sampling of area segments. Its use is advocated by virtue of the ease with which the standard errors of estimates can be obtained. The aim of convenience and simplicity in tabulations, e.g., the use of ranges to estimate standard errors, is in the background of most of the designs illustrated.

The style of the book is perhaps best expressed through an extensive use of survey materials and problem situations from the author's wide experiences. Instructions for serializing the frame and defining sampling units and instructions to the interviewers and on tabulation procedure are patiently and fully set down for each of the major types of design. As exercises, the examples should prove invaluable in leading students through all phases of an actual survey. They might also prove somewhat frustrating in a search for more general principles.

The general principles are introduced as needed in the particular examples and heavy cross-referring by page number is employed to supply continuity. Notions such as balanced thinning, thick zones and thin zones, "Cdus", primary areas, Masuyama's zigzag interval and ratio estimates in fine classes are scattered rather widely. One cannot escape the implication that a sampling craftsman should acquire competence with the widest variety of devices and precepts and then perhaps lean on intuition to pick the ones most relevant to the job at hand.

The statistical theory proper appears (in Part III) to have been carefully culled and tailored to sampling problems. The liberal use of rel-variances (coefficients of variation squared) and approximate expressions allows the complicated notation and algebra, usual in this sector, to be sidestepped. The concepts "estimate" and "bias" are derived from a random variable with its expected value by applying a functional transformation to both and noting the discrepancy, if any, between the expected value of the transformed random variable and the transformed expected value. Although these definitions-by-implication are specialized, they provide a convenient scheme for summarizing the more common problems of bias in sample surveys.

In short, the book goes deeply into specific examples with rich overtones of middle-range principles. As the author observes, it "stops short of complex treatments" and deserves supplementation by other sampling texts. Although there is some new material, the general emphasis is on conventional approaches.

C. H. Proctor (Raleigh, N.C.)

11503:

Pierson, Willard J., Jr. On the use of time series concepts and spectral and cross-spectral analyses in the study of long-range forecasting problems. J. Marine Res. **18** (1960), 112–132.

This is an interesting application of recently developed methods for the spectral analysis of vector-valued Gaussian processes. The formulae of N. R. Goodman [*On the joint estimation of the spectra, co-spectrum and quadrature spectrum of a two-dimensional stationary Gaussian process*, Sci. Pa. Engrg. Statist. Lab., N.Y. Univ. No. 10 (1957), unpublished] are used. The long-period fluctuations are found to be fairly coherent, and prediction problems are discussed. *W. Freiberger* (Providence, R.I.)

11504:

Janáč, Karel. Determination of correlation functions on the output of a generator of continuous random processes. Apl. Mat. **6** (1961), 25–35. (Czech. Russian and English summaries)

The generator on whose output there is a continuous stationary random signal with a given correlation function is realized in the paper by linear filter with constant coefficients and a transfer-function $F(p)$; on his input comes a white noise. An advantageous method of computing the correlation function on the output is presented, determined by means of the unilateral Laplace transform as impulse response of another filter with transfer function $Q(p)$. The method of computation of the transfer-function $Q(p)$ is given and conditions which ensure that its impulse response be a correlation function are determined. The transfer function $Q(p)$ may be simulated for example on an analogue computer and register directly the impulse response. From the consideration of restricting the class of possible functions $Q(p)$ expressed as ratios of two polynomials it follows that the polynomial in the numerator of the transfer-function has exactly one unit less degree than the polynomial in the denominator. The computational procedure derived is illustrated by examples.

J. Janko (Prague)

11505:

McFadden, J. A. The fourth product moment of infinitely clipped noise. IRE Trans. **IT-4** (1958), 159–162.

Let $\xi(t)$ be a stationary, ergodic, "sufficiently smooth" stochastic process. There is no known satisfactory method of computing the exact probability density, $P_0(\tau)$, of the periods between zeros of such a process, even under the assumption that $\xi(t)$ is Gaussian. Following previous papers [IRE Trans. **IT-2** (1956), 146–150; **IT-4** (1958), 14–24; MR **20** #4895, 4896] continued use is made of the statistical properties of the function $x(t)=\operatorname{sgn}\xi(t)$. The author computes the fourth product moment $w(\tau_1,\tau_2,\tau_3)=E[x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)]$, where $x(t)=\pm 1$, and the zeros of $x(t)$ obey a Poisson distribution. He discusses the value of this result as an approximation for the case when $x(t)=\operatorname{sgn}\xi(t)$, $\xi(t)$ Gaussian. *E. Reich* (Aarhus)

11506:

Daniels, H. E. Approximate solutions of Green's type for univariate stochastic processes. J. Roy. Statist. Soc. Ser. B **22** (1960), 376–401.

The author considers the equations governing the time development of either the probability density p or the moment generating function M of a Markov variate, and approximates these by neglecting higher differentials of $\log p$ or $\log M$. This is just the WKB method, recognised first as a general technique by G. Green in 1837, its classic application being the approximation of wave optics by geometrical optics. The author makes the point that there are reasons for expecting the method to be useful even when there is no obvious "small parameter" corresponding to wavelength in the optical application.

The method has the advantage over a diffusion approximation that relative errors in p are small—in particular, the approximate p will not assume positive values outside the domain of the variate. The author's justification is largely by example: he has some simple examples for which one can see that the exact and approximate solutions are very close in functional form, and some numerical examples, deliberately unfavourable, for which the exact and approximate solutions agree extremely well.

The first approximation is a nonlinear first-order partial differential equation, which can be solved in some cases. The author shows how higher approximations can be calculated, and adds some interesting observations, relating the approximate solutions in p and M, and demonstrating the relationship of the method to that of steepest descents.

P. Whittle (Cambridge, England)

11507:

Solomon, Herbert. (Editor). ★Studies in item analysis and prediction. Stanford Mathematical Studies in the Social Sciences, VI. Stanford University Press, Stanford, Calif., 1961. xii+310 pp. $8.75.

A set of 19 chapters, by different authors, under three headings: Part I, Item selection: multivariate normal structure; Part II, Item selection: several non-parametric situations; Part III, The classification statistic and its distribution. The papers will be reviewed separately.

NUMERICAL METHODS

See also A10944, A11113, A11157, A11160, A11237, A11238, A11241, A11242, A11280, 11480.

11508:

Moriguti, Sigeiti. Notes on the numerical convergence of iterative processes. Contributions to probability and statistics, pp. 309–321. Stanford Univ. Press, Stanford, Calif., 1960.

For Newton's method and certain other iterative processes applied with roundoff, the author determines the fixed points and cycles for various starting points.

J. Kiefer (Ithaca, N.Y.)

11509:

Greenberg, B. G.; Sarhan, A. E. Generalization of some results for inversion of partitioned matrices. Contributions to probability and statistics, pp. 216–223. Stanford Univ. Press, Stanford, Calif., 1960.

Formulas for inverses of matrices with diagonal blocks of multiples of identities and off-diagonal constant blocks.

J. Kiefer (Ithaca, N.Y.)

11510:

Duffin, R. J.; Schmidt, Th. W. An extrapolator and scrutator. J. Math. Anal. Appl. **1** (1960), 215–227.

This paper describes a method of extrapolation from equally spaced data by fitting polynomials of degree not exceeding a stated integer. The polynomials are fitted by the criterion of "discounted least squares": that is, the expression $\sum_{n=1}^{\infty} \theta^n[y_n - p(n)]^2$ is minimized, where y_n denotes the observed value for $x=n$, $p(x)$ is the fitted polynomial, and θ is a constant between 0 and 1 called the "discount factor". It is shown that the extrapolated value $y_0^* = \sum_{n=1}^{\infty} \theta^n q(n) y_n$, where

$$q(n) = -\sum_{k=0}^{m} (1-\theta^{-1})^k \binom{m}{k}\binom{x-1}{k-1},$$

if the polynomial is required to be of degree $m-1$ or less. It is also shown that the identity

$$y_0^* = \sum_{j=1}^{m} (-1)^{j+1} \binom{m}{j} (y_j + \theta^j \delta_j),$$

is satisfied, where $\delta_k = y_k^* - y_k$, and y_k^* is the value for $x=k$ extrapolated from $y_{k+1}, y_{k+2}, \cdots$, and it is proved that a single multi-step extrapolation gives the same result as successive single-step extrapolation. It is suggested that the process can also be used for "scrutation" (i.e., rejecting outlying observations).

T. N. E. Greville (Kensington, Md.)

11511:

Guest, P. G. ★Numerical methods of curve fitting. Cambridge University Press, New York, 1961. xiv+422 pp. $15.00.

This is a practical man's handbook of statistical and computational techniques for fitting observed data by polynomials and other functions. It contains an enormous amount of material in a condensed form, with many numerical examples. The intended audience is students and practitioners of physics, but it could serve others as well. The author is at the University of Sydney, Australia, and has written many papers in the field of the book.

The algorithms presented are designed primarily for desk calculators, and the preface expresses the faith that methods for automatic computers will be very similar. {The reviewer has equal faith that the similarity will decrease very rapidly with passing years.}

The book has 12 chapters, grouped into three parts (I. Single Variables; II. Regression Theory and the Straight Line; III. Polynomials and Other Curves). Part I gives statistical background on distributions (including normal, Poisson, binomial, etc.); moments; F-, t-, and χ^2-tests; etc. The theory follows the intuitive and formal approach used successfully by physicists and deplored by mathematicians. Thus, if $\varphi(t)$ is the Fourier transform of a frequency function $f(y)$, we are given a "proof" on pp. 22–23 that the inversion formula

$$f(y) = (2\pi)^{-1} \int_{-\infty}^{\infty} e^{-ity} \varphi(t)\, dt$$

always holds, without even a hint that it will ordinarily require considerable analysis to make the inversion formula meaningful or computable. Intuitive mathematics is valuable, but surely it should be labeled as such.

Practical statistics starts in Part II, with considerable discussion of linear regression problems where either one variable or both variables are subject to error.

Numerical analysis begins mainly in Part III, with six chapters: 7 (estimation of polynomial coefficients), 8 (standard deviations of the estimates), 9 (grouping of data), 10 (non-polynomial functions), 11 (several variables), and 12 (more examples). Matrix notation occurs occasionally. Doolittle methods for solving linear systems are discussed, as are Seidel's iteration {attributed to a "von Seidel"} and steepest-descent methods. The value of orthogonal polynomials is brought out. There are practical methods for dealing with missing data. Sampling the remaining material, we note a section on harmonic analysis and one on special methods for grouping data.

Throughout the book are perhaps 40 pages of reference tables for use with desk calculations. These are very detailed, but it is unlikely that many of these will be useful with automatic computers.

There is a bibliography of about 45 books and 160 articles at the end of the book, together with references to them at the end of each chapter. There is a modest index combining subjects and authors.

In summary, the author has brought into one handbook a large amount of formal statistics and desk-calculator numerical analysis dealing with his subject, together with many practical hints and numerical examples. The theory is unsophisticated, and the methods are not completely integrated. But the book will be a standard reference until some automatic-computer user replaces it.

G. E. Forsythe (Stanford, Calif.)

11512:

Nugeyre, J.-B. Étude de procédés d'inversion numérique de la transformation de Laplace-Carson. Chiffres **3** (1960), 101–116. (English, German and Russian summaries)

The author considers the first method of Papoulis [Quart. Appl. Math. **14** (1956), 405–414; MR **18**, 602] and some variants thereof for the numerical inversion of Laplace transforms $F(p) = \int_0^{\infty} e^{-pt} f(t)dt$ which use $F(p)$ for real values of p only. Papoulis' approximate inversion $f_n(t)$ is given by

$$f_n(t) = (4/\pi) \sum_{p=0}^{n} \lambda_{pn}(\arccos \exp(1-\sigma t)) F((2p+1)\sigma),$$

where $\lambda_{pn}(\theta)$ are independent of F. In the above formula, σ is a free parameter. After giving some estimates and numerical examples for the error $f - f_n$ the author suggests making σ dependent on t in such a way that $\sum_{p=0}^{n} \lambda_{pn}^2(\theta)$ assumes a relative minimum. Some estimates and numerical examples of this variant are then given. In the other variant, an approximate $A_n f(t)$ is given by $\sum_{k=0}^{n} a_{kn} F(b_{kn}/t)$, coefficients are chosen so that $f(t) = \sin t$ is approximated well, and functions which are finite Fourier sine transforms $\int_0^B g(u) \sin ut\, du$ are approximated by $\int_0^B g(u) A_n(\sin ut) du$

H. O. Pollak (Murray Hill, N.J.)

11513:

Poplavskiĭ. R. P. A possibility of improving convergence in direct methods of mathematical physics. Vyčisl. Mat. **6** (1960), 63–73. (Russian)

The author considers the choice of approximating functions for applying methods of Ritz, Trefitz, and the like to several classes of boundary-value problems.

A. S. Householder (Oak Ridge, Tenn.)

11514:

Albrecht, J. Formeln zur numerischen Integration über Kreisbereiche. Z. Angew. Math. Mech. **40** (1960), 514–517.

Formulas for the evaluation of integrals over the circle $x^2+y^2 \leq r^2$ are calculated which give high accuracy while requiring the values of the integrand at comparatively few points only. These points are vertices of a certain set of inscribed regular polygons. A table of the positions of the points and of the respective weight factors for various degrees of accuracy is calculated. The paper leans heavily on a paper by J. Albrecht and L. Collatz [same Z. **38** (1958), 1–15; MR **20** #432]. *W. Wasow* (Madison, Wis.)

11515:

Betts, D. D. The extension of Houston's method with application to Debye Θ_0. Canad. J. Phys. **39** (1961), 233–238.

The author gives some approximate integration formulas for the evaluation of integrals over the surface of the 3-dimensional unit sphere. His formulas are exact for polynomials, up to a certain degree, which have the same symmetry as a cube. He gives formulas which involve 9 and 15 points in a triangular section of 1/48th of the surface of the sphere. These formulas extend 3- and 6-point formulas given by Houston [Rev. Mod. Phys. **20** (1948), 161–165] and by Betts, Bhatia and Wyman [Phys. Rev. (2) **104** (1956), 37–42; MR **18**, 475].

These formulas are applied to calculate Θ_0, the Debye characteristic temperature at 0° K, for 22 crystalline solids. The 9 and 15 point formulas seem to give good results in these calculations except for the alkali metals lithium and sodium. *A. H. Stroud* (Madison, Wis.)

11516:

Grebenyuk, D. G. A generalization of the Gauss quadrature formula to the case of m variables. Izv. Akad. Nauk UzSSR. Ser. Fiz.-Mat. **1958**, no. 2, 69–76. (Russian. Uzbek summary)

This paper discusses approximate integration formulas for the m-dimensional cube of the form

$$\int_{-1}^{1}\cdots\int_{-1}^{1} f(x, y, \cdots, t)\,dx\,dy\cdots dt = \sum_{i=1}^{r} A_i f(x_i, y_i, \cdots, t_i) + R_r$$

which are exact for all polynomials of degree $\leq n$ in the m variables $x, y, \cdots, t$. He calculates only a few simple Cartesian product formulas for $m=3$, none of which are new; see, e.g., Hammer and Wymore, Math. Tables Aids Comput. **11** (1957), 59–67 [MR **19**, 323]. However the formulas are derived from a point of view which is different from that used by previous authors, and an expression for R_r is derived.

A. H. Stroud (Madison, Wis.)

11517:

Korobov, N. M. Properties and calculation of optimal coefficients. Dokl. Akad. Nauk SSSR **132** (1960), 1009–1012 (Russian); translated as Soviet Math. Dokl. **1**, 696–700.

The function

$$f(x_1, \cdots, x_s) = \sum_{m_1,\cdots,m_s=-\infty}^{\infty} C(m_1, \cdots, m_s) \exp\left[2\pi i(m_1x_1+\cdots+m_sx_s)\right]$$

is said to belong to the class E_s^α if $C(m_1, \cdots, m_s) = O((\overline{m}_1\cdots\overline{m}_s)^{-\alpha})$, where $\alpha>1$ and $\overline{m}_\nu=\max(1, |m_\nu|)$. Let $-R$ denote the error in the quadrature formula

$$\int_0^1\cdots\int_0^1 f(x_1, \cdots, x_s)\,dx_1\cdots dx_s = \frac{1}{p}\sum_{k=1}^{p} f\left(\frac{ka_1}{p}, \cdots, \frac{ka_s}{p}\right) - R.$$

The integers $a_\nu=a_\nu(p)$ $(\nu=1, 2, \cdots, s)$ are called optimal coefficients if the estimate $R=O(p^\alpha \ln^\beta p)$ holds for functions $f\in E_s^\alpha$, where β and the constant in the O term do not depend on p.

In a previous paper [Dokl. Akad. Nauk SSSR **124** (1959), 1207–1210; MR **21** #2848] the author established the existence of optimal coefficients for which $\beta=\alpha s$ and gave a method for finding them. In the present paper another method is given for calculating optimal coefficients; some of their properties which have applications both in numerical analysis and the theory of numbers are also derived. *A. H. Stroud* (Madison, Wis.)

11518:

Schröder, Johann. Fehlerabschätzung mit Rechenanlagen bei gewöhnlichen Differentialgleichungen erster Ordnung. Numer. Math. **3** (1961), 39–61.

Let a set of numbers y_k be given which approximate the solution $u^*(t)$ of $\dot{u}=F(t, u)$, $u(t_0)=u_0$ at equidistant points $t_k=t_0+kh$. The method used to obtain the y_k is irrelevant except that it should be of order ≤ 4. The author describes a computer program for finding corresponding error bounds which is based on an estimate proved elsewhere by the author {the reference given in the bibliography seems to be incorrect}. The program requires a considerable amount of preliminary mathematical analysis, but then seems to produce very satisfactory results. A number of numerical experiments is reported.

Walter Gautschi (Oak Ridge, Tenn.)

11519:

Hull, T. E.; Newbery, A. C. R. Integration procedures which minimize propagated errors. J. Soc. Indust. Appl. Math. **9** (1961), 31–47.

"Our concern in this paper will be with choosing the corrector formula which is best in the sense that the resulting propagated error (or, more precisely, a certain estimate of this error) is a minimum." The corrector formulas considered are of the form

$$a_k y_{n+k} + \cdots + a_1 y_{n+1} + a_0 y_n = h(\beta_k y'_{n+k} + \cdots + \beta_0 y_n').$$

"We shall consider the problem in several stages. We first obtain an estimate of the propagated error which will be used later to motivate our choice of formulas. We then describe a collection of corrector formulas which satisfy the essential requirement of stability, but which are at the same time of maximum degree (i.e., precision). Next we show which of our collection of stable, maximum-degree formulas can be expected to minimize our estimate of the propagated error. In this way we are led to a one-parameter family of 'best' formulas, the value of the parameter being in each case dependent on the particular differential equation being considered. Finally we apply our formulas to some special cases, and present some numerical evidence in support of our conclusions."

The authors show that one characteristic root is e^{hg} (where $g=\partial f/\partial x$); as for the others:

"Stability of the numerical method requires that these extraneous roots have magnitudes less than unity so that the corresponding components of the error will be negligible." {This criterion is open to debate when g is negative since then some of the extraneous roots could produce errors that would be greater than the solution being computed.}

The paper is well written and clearly recognizes the importance of stability, especially the futility of searching for high-accuracy formulas of order greater than $k+1$ (occasionally $k+2$), as has been done recently in the literature in several places.

The paper closes with a discussion of further possible research and includes a good bibliography.

R. W. Hamming (Stanford, Calif.)

11520:

Sangren, Ward C. ★Digital computers and nuclear reactor calculations. John Wiley & Sons, Inc., New York-London, 1960. xi+208 pp. $8.50.

As the author states in his preface, the primary objective of this book is to present to nuclear engineers and scientists an introduction to high-speed reactor calculations. Since the appearance of the basic reference *The elements of nuclear reactor theory* by Glasstone and Edlund [Van Nostrand, Princeton, N.J., 1952], the entire complexion of actual reactor-design calculations has changed as a result of the growth in speed and size of computing machines, and reactor-design calculations represent today a significant part of scientific computing time on modern computers.

The outline of the book by chapters is as follows: (1) Introduction; (2) Digital computers; (3) Programming; (4) Numerical analysis; (5) A code for fission-product poisoning; (6) Diffusion and age-diffusion calculations; (7) Transport equation—Monte Carlo; (8) Additional reactor calculations. In Chapter 1, the author reviews the tremendous parallel growth of high-speed computing machines and nuclear reactors, and their present interplay. In Chapter 2, an introduction and description of present-day computers is given. In Chapter 3, programming for computers is introduced. After some preliminary remarks (no proofs) about interpolation, numerical integration, matrices, etc., items which can be found in many well known texts on elementary numerical analysis, the author treats in Chapter 4 the more relevant problem of the numerical approximation of partial differential equations by difference equations, and their solution by means of iterative methods. Also, the treatment of interface conditions which arise naturally in heterogeneous reactors is given. In Chapter 5, a simple code for fission-product poisoning is followed from the physical and mathematical definitions through to the construction of a program in the Bell (Wolontis) system. In Chapter 6, the longest chapter, the author describes diffusion calculations, starting with steady-state criticality problems for reactors, to the solution of two- and three-dimensional multigroup diffusion equations. In Chapter 7, the S_n method of Carlson is described, along with the use of Monte Carlo methods for solving problems such as those encountered in shielding calculations.

{In his primary objective, the author does succeed. The reviewer feels, however, that some of the topics discussed are now somewhat dated. Nevertheless, the book is the only existing bridge between *The elements of nuclear reactor theory* [op. cit.] and present-day computing techniques in the reactor field. There are several important errors in defining the Young-Frankel and Peaceman-Rachford iterative methods.}

R. S. Varga (Cleveland, Ohio)

11521:

Moore, Robert H. A Runge-Kutta procedure for the Goursat Problem in hyperbolic partial differential equations. Arch. Rational Mech. Anal. **7** (1961), 37–63.

The subject of this paper is the development of a numerical procedure analogous to the Runge-Kutta method for the problem

$$u_{xy} = f(x, y, u, u_x, u_y)$$

$$u(x, 0) = \sigma(x), u(0, y) = \tau(y), \sigma(0) = \tau(0)$$

$$0 \leq x \leq a, 0 \leq y \leq b.$$

The passage from one to two dimensions makes the resulting formulas—and even more their derivation—too complicated to be summarized here. The calculation proceeds stepwise in a rectangular grid with mesh sides $\Delta x = h$, $\Delta y = \varphi h$. Assuming the values of u, u_x, u_y at (x_0, y_0), (y_0+h, y_0), $(y_0, y_0+\varphi h)$ to be known, formulas are given for the approximate determination of u, u_x, u_y at $(x_0+h, y_0+\varphi h)$. The calculation involves intermediate values in such a way that 18 possible different increments of the coordinates have to be considered simultaneously. With these increments a set of formulas is proposed, which, when written in matrix form, resembles structurally the ordinary Runge-Kutta formulas. The weight factors in these formulas, 270 in number, together with the 18 increments, are then determined from the requirement that the Taylor expansions of these formulas should agree with the differential equations for u, u_x and u_y through terms of order h^4, h^3, h^3, respectively. The resulting conditions are underdetermined and permit the imposition of some symmetry and positivity requirements. After considerable calculation a particular set of values for all parameters is proposed, which, in the end, produces a comparatively simple numerical scheme. The discretization error inherent in this method is $O(h^5)$ for sufficiently smooth data. This is proved by applying the author's results in same Arch. **6** (1960), 75–88 [MR **22** #5805].

W. Wasow (Madison, Wis.)

11522:

Frey, T.; Rózsa, P. Konvergenzschnelle des Differenzverfahrens der Poissonschen und der biharmonischen Differentialgleichung. I. Period. Polytech. Engrg. **4** (1960), 385–422.

Poisson's equation and the non-homogeneous biharmonic equation in a rectangle are each replaced by the simplest approximating finite-difference equation. The solutions, and in the biharmonic case also their second normal derivatives, are to vanish at the boundary. Green's functions for these problems can be explicitly constructed; for the differential equations by means of infinite Fourier series, for the discrete problems by finite trigonometric interpolation series. With the help of these series the discretization errors, i.e., the differences between the solution of the differential-equation problem and the corresponding difference problem, are very carefully estimated. The resulting explicit inequalities involve bounds on the right members of the differential equations

and the modulus of continuity of these members. In the biharmonic case the first derivatives of the right members and their moduli of continuity also enter.

W. Wasow (Madison, Wis.)

11523:

Lax, Peter; Wendroff, Burton. Systems of conservation laws. Comm. Pure Appl. Math. **13** (1960), 217–237.

To approximate a weak solution of the system $u_t = f(u)_x = Au_x$, with initial vector $u(x, 0) = \varphi(x)$, subject to a Rankine-Hugoniot relation $s[u]+[f]=0$ at a discontinuity propagated with speed s, the authors use the difference equation $\Delta u = \lambda \Delta g$. Here $\lambda = \Delta t/\Delta x$, $\Delta u = u(x, t+\Delta t) - u(x, t)$, and $\Delta g = g(x+\frac{1}{2}\Delta x) - g(x-\frac{1}{2}\Delta x)$, where for some vector-valued function $g(a, b)$ of the vectors a and b such that (i) $g(u, u) = f(u)$, $g(x+\frac{1}{2}\Delta x) = g(u(x+\Delta x, t), u(x, t))$. Condition (i) guarantees that truncation errors are $O(\Delta^2)$. The special choice

$$2g(a, b) = f(a)+f(b)+\lambda A^2(b-a)+Q(a, b)\cdot(b-a),$$

where the "artificial viscosity" matrix Q is symmetric and $Q(a, a)=0$, yields a truncation error that is $O(\Delta^3)$. For $\lambda|\mu|_{\max} \leq 1$, where the μ are eigenvalues of A, the difference equation satisfies von Neumann's condition of stability. For scalar u the authors describe conditions for and a choice of Q which can be proved to yield approximations to a steady-state step-function solution by numerical functions in which the discontinuity will be spread over three successive mesh-points. For this Q von Neumann's stability condition is satisfied if $\lambda\mu \leq .78$. For vector u an appropriate Q is also described. An examination of the "tail ends" of the steady-state solution yields an explanation for overshoots and oscillations observed near computed shocks. The special case of one-dimensional flow is discussed, and results of computations of flows with shocks, contact discontinuities, and rarefaction waves are presented. *J. H. Giese* (Aberdeen, Md.)

11524:

Nandy, K. P. Multiquadrant graphs for functions of n-variables. J. Sci. Engrg. Res. **3** (1959), 123–129.

A function of n variables may be written in the form

$$F = \cdots\psi_2[\psi_1\{\phi\times\phi_1+\chi\}\times\xi+\eta]\cdots$$

where $\phi, \phi_1, \xi, \chi, \eta, \cdots$ are each a continuous function of one independent variable. A sheet of squared paper is divided into as many quadrants as there are independent variables. Then the four fundamental operations (addition, subtraction, multiplication and division) are performed plotting sets of curves in the quadrants. The graph for F is obtained step by step. The preparation of a multi-quadrant graph is illustrated for the equation

$$F = \left\{\frac{V+(\alpha x+y^2)^{1/2}}{r^2}+10\sin\tfrac{1}{2}\beta\right\}\log r^3.$$

D. Mazkewitsch (Cincinnati, Ohio)

11525:

Jokl, Evžen. Zur Abschätzung der Fehler bei den Rechnungen nach Nomogrammen. Apl. Mat. **6** (1961), 68–72. (Czech. Polish and German summaries)

The paper gives formulas for evaluation of errors of calculations by means of nomograms. Formula (1) is for grid charts with a curvilinear answer family, formula (2) for any grid chart. Formula (3) is for alignment charts in which the answer scale lies between the two others; formula (4) is for the opposite case. Formula (5) is applicable in some cases when the function to be nomographed is of the form $h_1f_3+h_2g_3+h_3=0$. Formula (7) gives more accurate evaluations than (3) and (4). The geometric structures in each case are shown.

D. Mazkewitsch (Cincinnati, Ohio)

11526:

Gadd, D. A construction for the graphical solution of spherical triangles. Math. Gaz. **43** (1959), 283–288.

In this paper the construction given by the author for the graphical solution of a spherical triangle, by a diagram in a plane containing two of the apices of the triangle and the centre of the sphere, is only available for those cases in which accuracy is not required. The author indicates some of the possible applications of the method.

M. Piazzolla-Beloch (Ferrara)

11527:

Fox, L. ★Tables of Weber parabolic cylinder functions and other functions for large arguments. National Physical Laboratory Mathematical Tables, Vol. 4. Department of Scientific and Industrial Research. Her Majesty's Stationery Office, London, 1960. iii+40 pp. 12s. 6d.

The most important table in this volume is table six. This consists of values to eight decimals of the four auxiliary functions f, g, ϕ and ω which are needed to determine the values of the Weber functions for large values of x. These tables cover the region $1/x = 0(0.005)0.1$ and $a = -10(1)10$, together with all the necessary aids to interpolation in both a and $1/x$, and they form an extension for large values of x of some of the tables already given by J. C. P. Miller [*Tables of Weber parabolic cylinder functions, giving solutions of the differential equation* $d^2y/dx^2+(\frac{1}{4}x^2-a)y=0$, Her Majesty's Stationery Office, London, 1955; MR **17**, 1012].

The other contents of the volume include a table of the exponential integrals $e^{-x}Ei(x)$ and $-e^{-x}Ei(-x)$, for $1/x = -0.1(0.001)0.1$, to ten decimals with modified second differences for interpolation. There are also tables of the sine and cosine integrals to ten decimals for $1/x = 0(0.001)0.1$ auxiliary functions for the Airy integrals to ten decimals for $z = (3/2)x^{-3/2} = -0.05(0.001)0.05$, and tables of various gamma functions for large values of x, all with full provision for interpolation.

This work is well up to the usual high standard attained in this series and it should prove of great interest to all serious students of numerical analysis.

L. J. Slater (Cambridge, England)

COMPUTING MACHINES

See also A10906, 11520, 12005, 12006, 12015, 12016.

11528:

Bartee, Thomas C. ★Digital computer fundamentals. McGraw-Hill Book Co., Inc., New York-London-Toronto, 1960. ix+342 pp. $6.50.

This is a quite elementary and relatively superficial account of digital computer technology. The chapters are:

(1) Operations with computers both desk and automatic. (2) Programming. (3) Number systems. (4) Basic logical circuits. (5) Logical design. (6) The arithmetic element. (7) The memory element. (8) Input-output devices. (9) The control element.

The chapter on number systems is entirely arithmetical in character and does not seek to show the underlying mathematical principles involved. Similarly the chapter on logical design treats Boolean algebra simply as a tool for circuit design and does not hint at its wider connections. Such circuits as appear are purely schematic and will not help practical designers, although the book has an up to date flavour in that it concentrates on transistor-diode-core elements and more-or-less ignores the older hard tube elements.

Each chapter ends with a useful set of examples and a bibliography. The examples are useful although for the most part elementary, but the bibliographies are incomplete and give a most imperfect view of the subject. As an elementary introduction for students the book may be of value, but it contains nothing of interest to mathematicians. *A. D. Booth* (London)

11529:

Línek, A.; Novák, C. Structure factor computing machine. Czechoslovak J. Phys. **10** (1960), 397–406. (Russian summary)

A description is given of a special single-purpose machine which computes, for a preassigned set of integer triples h, k, l, the functions $\sum_j f_j \cos \frac{2\pi}{1024}(hx_j+ky_j+lz_j)$, where f_j, x_j, y_j, z_j are given. The machine is designed for installation in a crystallographic laboratory where such functions find important application.

H. A. Hauptman (Washington, D.C.)

11530:

Sherman, P. M. Table look-at techniques. Comm. ACM **4** (1961), 172–173, 175.

In table look-at, tabular values for random arguments are retrieved by a single access to memory. The basic method is to load the argument into an index register and carry out a fetch-from-store construction, modified by the index register. Special features in the instruction repertory of a machine (e.g., indirect addressing, use of multiple index registers), can be turned to advantage. Using the FAP assembly language for the IBM 7090, short sequences of instructions illustrate table look-at applied to integer conversion, pattern recognition, many-way switching, sorting and selection of matrix elements.

C. C. Gotlieb (Toronto)

MECHANICS OF PARTICLES AND SYSTEMS

See also A11196, A11215.

11531:

Landau, L. D.; Lifshitz, E. M. ★Mechanics. Course of Theoretical Physics, Vol. 1. Translated from the Russian by J. B. Sykes and J. S. Bell. Pergamon Press, Oxford-London-New York-Paris; Addison-Wesley Publishing Co., Inc., Reading, Mass.; 1960. • vii + 165 pp. $6.50.

This is an excellent translation of the Russian original [*Mehanika*, Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1958] which has been reviewed already [MR **21** #985]. Misprints have been corrected; printing, paper and binding are excellent. *N. L. Balazs* (Princeton, N.J.)

11532:

Blaschke, Wilhelm. Anwendung dualer Quaternionen auf Kinematik. Rev. Mat. Cuyana **2** (1956), 61–77.

A later version of this paper is given in Ann. Acad. Sci. Fenn. Ser. A I no. 250/3 (1958) [MR **19**, 1099].

11533:

Gaiduk, V. I. On the energy variation of a particle or system under the action of external forces. Radiotehn. i Elektron. **4** (1959), 1513–1526 (Russian); translated as Radio Engrg. and Electronics **4**, no. 9, 158–180.

L'auteur considère une particule ou un système de particules en mouvement dans un champ de forces en général non conservatif, c'est-à-dire, fonction du temps. Il montre que l'énergie totale peut toujours être déduite de la solution d'un système d'équations différentielles du premier ordre (problème de Cauchy) sans qu'il soit nécessaire, au préalable, d'exprimer les coordonnées en fonction du temps. Lorsque les forces extérieures sont faibles, on peut faire appel à des méthodes connues pour construire une solution approximative.

L'article comprend quatre parties; les trois premières sont théoriques, la quatrième est une application. Dans la première partie, il s'agit d'une particule unique, ou d'un système à un seul degré de liberté. La solution d'une équation différentielle du second ordre est ramenée à celle d'un système de deux équations différentielles du premier ordre, ce dont l'auteur montre les avantages. Dans la second partie, l'étude du mouvement d'un système de particules est conduite à partir des équations de Lagrange et de la théorie d'Hamilton-Jacobi. On parvient naturellement à un système canonique du premier ordre, lorsque les forces extérieures dépendent d'un potentiel scalaire. La troisième partie se rapporte aux relations énergétiques des systèmes conservatifs, soit à l'approximation newtonienne, soit à partir des expressions relativistes du lagrangien et de l'hamiltonien. La quatrième partie est une application des méthodes précédemment exposées à la résolution approximative d'une équation différentielle du second ordre, à laquelle on parvient en étudiant le mouvement des électrons dans un tube nommé strophotron.

P. Poincelot (Issy-les-Moulineaux)

11534:

Balogh, A. Einiges über die Eigenschaften der charakteristischen Gleichung bei Torsionsschwingungen. Acta Tech. Acad. Sci. Hungar. **32** (1961), 429–440. (English, French, and Russian summaries)

A simple scheme is devised for evaluating numerically the characteristic determinant for the torsional oscillations of an n-mass system. There are a number of misprints.

B. R. Seth (Madison, Wis.)

11535:

Val'dman, A. [Valdmanis, A.]. Forced oscillations of a system with an elastic characteristic of five segments.

Latvijas PSR Zinātnu Akad. Vēstis **1959**, no. 10 (147), 51–58. (Russian. Latvian summary)

Es wird ein Schwinger von einem Freiheitsgrad untersucht, der durch harmonische Erregerkräfte zu erzwungenen Schwingungen angeregt wird. Die Rückführfunktion des Schwingers wird als bereichsweise linear mit insgesamt 5 Bereichen angenommen. Das nichtlineare Problem wird näherungsweise als linear behandelt, wobei der lineare Ersatzkoeffizient nach der Methode der kleinsten quadratischen Abweichung bestimmt wird. In der Arbeit werden die Beziehungen zwischen Amplitude und Frequenz in impliziter Form für viele Sonderfälle angegeben, darunter auch für Schwinger mit unsymmetrischen Rückführfunktionen. Lediglich für ein System mit hartem Anschlag ist eine Schar von Resonanzkurven ausgerechnet worden.

K. Magnus (Stuttgart)

11536:

Senik, L. M. Determination of a function characterizing the dispersion of energy of an oscillating system. Akad. Nauk Ukrain. RSR. Prikl. Meh. **6** (1960), 40–45. (Ukrainian. Russian and English summaries)

The oscillating system is of the form $\ddot{x}+\omega_0^2 x=\varepsilon f(x,\dot{x},\varepsilon)$. The function "characterizing the dispersion of energy" is the function f on the right-hand side. Suppose that the amplitudes and periods of the oscillations of the system can be observed. The problem posed is then the determination of a function f. With various assumptions the Krylov-Bogoliubov method of averaging is used to obtain an expression for a function f.

J. P. LaSalle (Baltimore, Md.)

11537:

Kuzmak, G. E. The motion of an axially symmetric solid in space about a stationary point under the action of moments slowly varying with time. Dokl. Akad. Nauk SSSR **132** (1960), 549–552 (Russian); translated as Soviet Physics. Dokl. **5**, 526–529.

Consider a gyro of revolution having a stationary point on its axle and being under the action of moments about the moving rectangular coordinate axes, of which one axis is the axle of the gyro, where the moments are depending on the slow time $\tau=\varepsilon t$ (ε is a small parameter, t is the time) and the angle of nutation θ. The equations of motion of the gyro are given in the form

$$\frac{d^2\theta}{dt^2}-\lambda^2\sin\theta\cos\theta+r(\tau)\lambda\sin\theta+F(\tau,\theta)+\varepsilon f(\tau)\frac{d\theta}{dt}=0,$$

$$\frac{d}{dt}[\lambda\sin^2\theta+r(\tau)\cos\theta]+\varepsilon[f(\tau)\lambda\sin^2\theta-r'(\tau)\cos\theta]=0,$$

where λ represents the precession velocity.

The author explains the process of obtaining an asymptotic solution of the equations by applying the method that he has developed in his other article treating a differential equation of the form $d^2y/dt^2+\varepsilon f(\tau,y)dy/dt+F(\tau,y)=0$ [same Dokl. **120** (1958), 461–464; MR **21** #169].

K. Yamada (Nagaoka)

11538:

Sagirow, P. Zur Frage der Fehlerabschätzung beim Verfahren der harmonischen Balance. Z. Angew. Math. Mech. **40** (1960), 456–463. (English and Russian summaries)

Wenn man die nichtlineare Differentialgleichung $L(D)x+f(x)=0$ mit dem linearen Differentialoperator $L(D)$ nach der Methode der harmonischen Balance (identisch mit der Methode der Beschreibungsfunktion) untersucht, dann bekommt man in bequemer Weise Näherungslösungen für Frequenz und Amplitude, der im System existierenden periodischen Lösungen. Verallgemeinert man nun den Ansatz für x durch Mitnahme höherer Glieder des Fourieransatzes, dann kann aus der Restreihe der Fehler der 1. Näherung abgeschätzt werden. Unter nicht sehr einschränkenden Voraussetzungen läßt sich zunächst eine Abschätzung für den Fehler der Frequenzbestimmung gewinnen. Damit kann dann auch der Amplitudenfehler eingegrenzt werden. Jedoch ist die Genauigkeit für die Abschätzung der Amplitude im allgemeinen erheblich schlechter, als bei Bestimmung der Frequenz. Am Beispiel einer Differentialgleichung 3. Ordnung wird das Verfahren demonstriert.

K. Magnus (Stuttgart)

11539:

Tabarovskiĭ, A. M. On the stability of motion of a heavy gyroscope on gimbals. Prikl. Mat. Meh. **24** (1960), 572–574 (Russian); translated as J. Appl. Math. Mech. **24**, 850–855.

Es wird die Stabilität spezieller permanenter Drehungen eines kardanisch gelagerten Kreisels untersucht, bei dem die Achse des äußeren Kardanringes einen beliebigen Winkel mit der Vertikalen bildet. Reibungskräfte werden vernachlässigt. Nach dem Vorbild von Četaev wird aus den Eigenschaften einer Ljapunovschen Funktion auf die Stabilität geschlossen. Dabei zeigt sich, daß Drehungen um die vertikal stehende Rotorachse nur stabil sein können, wenn der Schwerpunkt des aus dem Rotor und dem inneren Kardanrahmen gebildeten Systems unterhalb des Schnittpunktes der Kardanachsen liegt. Der Einfluß der Schräglage α der äußeren Kardanachse in den Grenzen $0\leq\alpha\leq\frac{1}{2}\pi$ wird untersucht.

K. Magnus (Stuttgart)

11540:

Bottema, O. Über die Stabilität der Präzessionsbewegung eines symmetrischen Kreisels. Ing.-Arch. **29** (1960), 425–427.

Auf einen symmetrischen Kreisel möge ein äußeres Moment wirken, das von einem nur von dem Eulerwinkel ϑ abhängigen Potential V abhängt. Gesucht werden reguläre Präzessionsbewegungen, d.h., Bewegungen mit $\vartheta=\vartheta_0=\text{const}$. Die Stabilität dieser Präzessionsbewegungen, die für jeden Wert von ϑ und für jeden Wert der Rotation um die Figurenachse existieren, wird untersucht. Es zeigt sich, daß die Stabilität wesentlich von dem Vorzeichen von $(dV/d\vartheta)_{\vartheta_0}$ und $(d^2V/d\vartheta^2)_{\vartheta_0}$ abhängt. Setzt man $V=mgh$, so wird man auf die bekannten Stabilitätsbedingungen für den schweren symmetrischen Kreisel geführt.

K. Magnus (Stuttgart)

11541:

Krementulo, V. V. Stability of a gyroscope having a vertical axis of the outer ring with dry friction in the gimbal axes taken into account. Prikl. Mat. Meh. **24** (1960), 568–571 (Russian); translated as J. Appl. Math. Mech. **24**, 843–849.

Es wird die Stabilität eines kardanisch gelagerten Kreisels mit symmetrischem Rotor untersucht, bei dem

die äußere Kardanachse vertikal steht. Als dämpfende Momente werden Coulomb'sche Reibungsmomente um beide Kardanachsen vorausgesetzt. Diese Voraussetzung macht es möglich, die gestörten Bewegungen bereichsweise nach den Ljapunovschen Verfahren abzuschätzen, indem aus bekannten Integralen der Differentialgleichungen definite quadratische Formen der Systemvariablen gebildet werden, deren totale zeitliche Ableitung verschwindet. Als Ergebnis folgt, daß Stabilität der vertikalen Lage der Rotorachse nur möglich ist, wenn der Schwerpunkt des Systems unter dem Unterstützungspunkt liegt. Dieses Ergebnis läßt sich auch noch verallgemeinern auf Fälle, in denen die Rotorachse nicht vertikal steht. Nur muß dann zusätzlich gefordert werden, daß das entstehende Schwerемoment seinem Betrage nach kleiner als das maximal mögliche Reibungsmoment ist. *K. Magnus* (Stuttgart)

11542:

Roĭtenberg, Ya. N. On the motion of gyroscopic devices under the influence of random forces. Prikl. Mat. Meh. **24** (1960), 463–472 (Russian); translated as J. Appl. Math. Mech. **24**, 677–690.

Bei der Untersuchung des Verhaltens von Kreiselgeräten auf schlingernden Schiffen hat man bisher die Schlingerbewegungen als sinusförmig angenommen. Nachdem nunmehr Sveshnikov durch Messungen nachgewiesen hat, daß die Schlingerbewegung eines Schiffes eine stationäre Zufallsgröße mit bestimmter Korrelationsfunktion und Spektraldichte ist, wurden diese Ergebnisse den vorliegenden Untersuchungen zugrunde gelegt. Der Verf. untersucht 3 Kreiselgeräte: einen Schlingerkreisel mit Stützmotor, ein Kreiselpendel und einen 2-Kreisel-Kompaß. In jedem der drei Fälle werden zunächst die interessierenden Kreiselwinkel als Funktionen des Schlingerwinkels in Operatorform ausgerechnet. Aus den Operatoren wird sodann unter Verwendung der Spektraldichte der Schlingerbewegung in bekannter Weise der quadratische Mittelwert für die Winkel als zweiseitig unendliches Integral über die Frequenz ausgerechnet. Die Integrale lassen sich auswerten und ergeben explizite Ausdrüke für die gesuchten Mittelwerte. An Zahlenbeispielen wird die Größe der Fehler für die drei Geräte gezeigt. Im Falle des Kreiselkompasses wird dabei auch ein Ausdruck für die interkardinalen Schlingerfehler angegeben. *K. Magnus* (Stuttgart)

11543:

Blaquière, A. ★Mécanique non linéaire: Les oscillateurs à régimes quasi sinusoïdaux. Mém. Sci. Math., Fasc. 141. Gauthier-Villars, Paris, 1960. 137 pp. 27 NF; $5.75.

The central theme of this monograph is a method that has been "discovered" by numerous engineers and has been given many different names. It is essentially the Krylov-Bogolyubov method of harmonic balance derived from their method of averaging [N. Krylov and N. Bogolyubov, *Vvdenie v nelineĭnuyu mehanika*, Kiev, 1937; English translation, Princeton Univ. Press, Princeton, N.J., 1943; MR **4**, 142; N. N. Bogolyubov and Yu. A. Mitropol'skiĭ, *Asimptotičeskie metody v teorii nelineĭnyh kolebaniĭ*, Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1958; MR **20** #6812; N. Minorsky, *Introduction to nonlinear mechanics*, J. W. Edwards, Ann Arbor, Mich., 1947; MR **8**, 583]. American and British engineers call it the method of describing functions or the method of equivalent linearization. The nonlinearities of a system are replaced by suitable linear terms (the describing function). This equivalent linearization is attractive to engineers, for now they can apply their knowledge of linear systems. It allows them to use frequency-response methods for the study of nonlinear systems. Krylov and Bogolyubov confined themselves to the application that can be justified: small nonlinearities and an autonomous linear part. Lacking any general analytic tool for handling nonlinear problems, the engineer applies the method with great abandon and finds the method useful even though he knows that at times it gives the wrong answer.

This monograph is a satisfactory introduction to the method for engineers, although at times the author obscures the central idea. Chapter I deals with free oscillations, Chapter II with forced oscillations and Chapter III with systems excited by white noise. There are many examples illustrating the application of the method to a variety of problems.

J. P. LaSalle (Baltimore, Md.)

11544:

Blaquière, Austin. Extension de la méthode de Nyquist aux systèmes bouclés non linéaires. Cahiers de Phys. **14** (1960), 301–330.

This paper is a discussion of the method of harmonic balance (describing functions) applied to nonlinear systems with a feed-back loop [see #11543]. In particular, the method is used to discuss the stability of a nuclear reactor. *J. P. LaSalle* (Baltimore, Md.)

11545:

Rosenberg, R. M. On normal vibrations of a general class of nonlinear dual-mode systems. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 275–283.

The system consists of two oscillators which are attached to fixed points by springs and connected by a third spring, each spring force being proportional to the kth power of the deflection. For motion in a straight line this gives a system in which the acceleration of each oscillator is a homogeneous function of the two displacements, and there are normal vibrations in which the displacements remain in a constant ratio, which may have one of several values. These modes, and their stability, are discussed; the discussion of stability involves somewhat speculative simplifications.

T. M. Cherry (Melbourne)

STATISTICAL THERMODYNAMICS AND MECHANICS

See also A11258, 11520, 11662a–b, 11777, 11782, 11783, 11789, 11943.

11546:

von Roos, Oldwig. Formal solution of Liouville's equation. J. Mathematical Phys. **1** (1960), 107–111.

11547:

Teitler, S.; Wallis, R. F. Liouville equation and the

resolvent formalism. J. Mathematical Phys. **1** (1960), 372–377.

Van Hove's resolvent formalism is applied to the time evolution of the distribution function of a classical weakly coupled gas which has been investigated by Prigogine and Balescu by means of their diagram representation. On account of the compactness of the resolvent formalism, higher-order contributions to the relaxation phenomena are also considered. The authors state that their result confirms Van Hove's procedure about diagonal transitions. Any careful consideration, however, is not given to the analysis of the non-diagonal transitions. Accordingly, this may not be a definite answer to Prigogine and Balescu's objection against the diagonal singularity; it is only when the disconnected diagrams play no role that the diagonal transitions may become dominant, for the contribution of the non-diagonal transition corresponding to disconnected diagrams is much larger (by a factor N) than that of the diagonal transition. *R. Kubo* (Tokyo)

11548:

Caspers, W. J. On the theory of spin-spin relaxation. I, II, III. Physica **26** (1960), 778–797, 798–808, 809–824.

The spin-spin relaxation is treated theoretically using the method of Kubo and Tomita in the limit of high temperatures. In part I it is shown that the relaxation function satisfies a certain integral equation, from which the author obtains the asymptotic form of the function in a simple exponential function. The general expression of the relaxation is thus derived. For more complicated systems which contain many kinds of ions the relaxation function will be a sum of exponential functions. The field dependence of the relaxation time is discussed. In part II the theory is applied to the example of the ammonium cupric Tutton salt, assuming dipolar and exchange interaction between the spins. The results of numerical calculation are compared with the experiments. The author also compares the present theory to the older theories. The treatment of part I is extended in part III to such spin systems in which a certain field other than the Zeeman field is present for each constituent spin.

R. Kubo (Tokyo)

11549:

Hill, Terrell L. ★Statistical mechanics: Principles and selected applications. The McGraw-Hill Series in Advanced Chemistry. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1956. xiii+432 pp.

The principles of statistical mechanics are given in a brief, advanced presentation. The application of these principles to the imperfect gas, condensation, distribution functions and liquid state, and to lattice theories is given in detail. The book is restricted to equilibrium phenomena and, although published five years ago, is still fairly well up-to-date. *J. Ross* (Providence, R.I.)

11550:

Andrews, Frank C. The Green's function of the Liouville equation. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 475–485.

The author studies various types of Green's functions of the Liouville operator: retarded Green's function of the complete Liouville equation, Green's function of the time-independent Liouville equation (appearing in scattering problems), etc. The solution of the equations is uniformly done by calculating formally the inverse Liouville operator in Fourier space. The method is applied to the derivation of a conservation theorem which provides a link between Prigogine's and Bogoliubov's theories of irreversible processes. *R. Balescu* (Brussels)

11551:

Richardson, J. M. The hydrodynamical equations of a one component system derived from nonequilibrium statistical mechanics. J. Math. Anal. Appl. **1** (1960), 12–60.

The paper consists of a derivation of hydrodynamical equations from a microscopic model. As for the dynamics of the model, it is assumed that the molecules are all alike and interact (classically) with central forces, possessing no other internal structure. Of the statistical ensemble it is assumed that it is "hypercanonical" at an initial time $t=0$ in the following sense: Given a relatively small number of quantities of macroscopic interest, α_l, the logarithm of the distribution function is to be a linear combination of the form $-\sum_l b_l\alpha_l$, where the b_l are chosen so that the mean values $\langle\alpha_l\rangle$ have preassigned values. For such a hypercanonical ensemble it is then well-known that the "observable entropy", i.e., the Gibbs entropy of an "equivalent" hypercanonical ensemble at any other time t, $\bar{S}(t)>\bar{S}(0)=S$, S being a constant of the motion. {It is to be noted that this inequality also holds for $t<0$; thus the hypercanonical ensemble is not a completely attractive model of the irreversible non-equilibrium macroscopic physical system.} The general theory is applied to special limiting situations, the low-density limit, and the near-equilibrium case.

P. G. Bergmann (Syracuse, N.Y.)

11552:

Klimov, V. N. The kinetic equation for mixtures. Teor. Veroyatnost. i Primenen. **2** (1957), 266–274. (Russian. English summary)

A derivation is given of the kinetic equation for the number-density of particles entering a medium in small quantities when their interactions may be neglected. The transition probabilities, being determined only by the state of the medium, are expressed in terms of the density of the particles, their temperature, and their hydrodynamical velocity, which are given for each material component as an arbitrary function of time and place. The scattering is considered elastic. The kinetic equations for the two cases of neutrons and charged particles moving in a plasma are derived from the general equation.

K. C. Westfold (Clayton)

11553:

Sengers, J. V.; Cohen, E. G. D. Statistical mechanical derivation of the generalized Boltzmann equation for a fluid consisting of rigid spherical molecules. Physica **27** (1961), 230–244.

11554:

Cohen, E. G. D. On the connection between various derivations of the Boltzmann equation. Physica **27** (1961), 163–184.

11555:

Verlet, L. On the theory of classical fluids. Nuovo Cimento (10) **18** (1960), 77–101. (French and Italian summaries)

A discussion, one of many in the current literature, of the use of diagram techniques for the manipulation of the cluster expansion of the distribution function of non-ideal fluids. Depending on the kind of diagrams retained, various integral equations for the distribution function of a pair of molecules can be obtained and solutions can be attempted. This approach is in vogue but considered by this reviewer as a tangential attack on the most difficult many-body problem of the equilibrium theory of fluids.

J. Ross (Providence, R.I.)

11556:

Meeron, Emmanuel. Theory of classical fluids and the convolution approximation. Note on papers by Tohru Morita. Progr. Theoret. Phys. **24** (1960), 588–592.

The object of this paper is to establish an equivalence between the new improved form of Morita's "hypernetted chain" approximation [Progr. Theoret. Phys. **23** (1960), 829–845; MR **22** #6122] and the "nodal expansion" approximation which Meeron introduced a few years ago. A table of formulas is given which enables most, but not all, of Meeron's results to be translated into Morita's notation. Meeron points out that a limiting form of both his own and Morita's revised approximation, which he calls the "convolution" approximation, satisfy Kirkwood's superposition principle. *H. S. Green* (Adelaide)

11557:

Dupree, Thomas H. Dynamics of ionized gases. Phys. Fluids **4** (1961), 696–702.

Rostoker and Rosenbluth [Phys. Fluids **3** (1960), 1–14; MR **22** #4340] applied the Mayer cluster-integral technique to the solution of the moment equations of the Liouville equation for a plasma. Dupree gives the details of how to obtain the two-particle distribution function in terms of the one-particle function, neglecting higher-order correlations, or, equivalently, neglecting powers of N^{-1} higher than the first in the Liouville moment equations, N being the number of particles in a Debye sphere. He shows that the two-particle distribution function can be obtained by solving two inhomogeneous Vlasov equations, the inhomogeneities being the one-particle distribution functions. He concludes that, when the Vlasov equation predicts plasma instability, the neglected two-particle correlations (collision term) grow exponentially, vitiating the Vlasov approximation. He states that preliminary calculations indicate that this effect tends to damp out the instability. *G. E. Backus* (La Jolla, Calif.)

11558:

Trubicyn, V. P. The equation of state of metals by a statistical method. Fiz. Tverd. Tela **2** (1960), 898–902 (Russian); translated as Soviet Physics. Solid State **2**, 820–823.

Author's summary: "On the basis of a modified statistical method, an approximate equation of state is found for metals for pressures from zero to a million atmospheres and $T=0°$ K. In the statistical expression for the energy, the electron density used for the ion core is that found by the Hartree-Fock method and approximated by an exponential function. An explicit analytical expression is obtained for the dependence of the energy and pressure on the volume for metals."

G. Weiss (Washington, D.C.)

11559:

Brout, R. Statistical mechanical theory of ferromagnetism. High density behavior. Phys. Rev. (2) **118** (1960), 1009–1019.

Author's summary: "The partition function of the Ising model of ferromagnetism is examined in the limit of high density in the anticipation that in the limit of infinite density one recovers the Weiss molecular field. The formal parameter of expansion is $1/z$ where z is the number of spins in the range of the exchange potential (not restricted to nearest neighbor interactions). In the absence of long-range order, only ring diagrams in the cluster expansion contribute. These give a divergence in the specific heat at $kT_c=\sum_{j\neq i} v_{ij}$ where v_{ij} is the exchange potential. This is the molecular field value for the Curie point T_c. In the presence of a magnetic field the partition function is evaluated for fixed magnetic moment M in the same approximation, M being determined by minimization. This results in a susceptibility differing from the molecular field theory and hence an inconsistency in the theory.

"The inconsistency is traced back to the observation that the acceptance of ring diagrams is equivalent to the gaussian model of Kac and Berlin which violates the sum rule $\sum_{i=1}^{N}\mu_i^2=N$. Here μ_i is the 'spin' per particle and N is the total number of particles. This condition is reinstated by insuring the sum rule. The result leads to the spherical model. Thus, a consistent high-density approximation to the Ising model is the spherical model. Below the Curie point or for fixed magnetic field, M is again held fixed and only the Fourier components of the spin density with nonvanishing wave vector are 'sphericalized'. The result leads to a physically acceptable model which becomes the molecular field theory at low temperatures or high fields and deviates in $O(1/z)$ in general. Formally, the results are simply expressed in terms of a temperature-dependent Weiss field. These results differ from the ordinary spherical model which is physically unacceptable below the Curie point. However, a molecular field modification of the spherical model due to M. Lax yields the same result when properly interpreted.

"It is shown that the above results are also valid (to the same approximation) in the quantum mechanical Heisenberg model, for temperatures above the Curie point." An error of reasoning in the neighborhood of $T=T_c$ is corrected in a later paper [#11560].

S. Sherman (Detroit, Mich.)

11560:

Brout, R. Statistical mechanics of ferromagnetism; spherical model as high-density limit. Phys. Rev. (2) **122** (1961), 469–474.

A proof is given of a conjecture which was set forth in a previous work [R. Brout, #11559]. If z is the number of spins interacting with a given one (the interaction is not restricted to nearest neighbors), the diagrams occurring in the Ising and Heisenberg models can be classified in such a way that the dependence on z of these classes is shown. The spherical model of Berlin and Kac [Phys.

Rev. (2) **86** (1952), 821–835; MR **14**, 232] corresponds to the summation of cycle diagrams with certain correction for excluded volume; the class of diagrams which is omitted in the spherical model is of higher order in $1/z$. This means that the spherical model is in fact a first approximation to the Ising model.

J. Peretti (Montpellier)

11561:

Maradudin, Alexei A.; Weiss, George H.; Jepsen, Donald W. Nonequilibrium processes in isotopically disordered crystals. J. Mathematical Phys. **2** (1961), 349–369.

The aim of this paper is to find the time evolution, due to impurity scattering, of the normal modes (n.m.) of an ideal crystal containing two isotopes with masses M_1 and M_2 The n.m. refer to a virtual lattice with mass $M = pM_1+(1-p)M_2$, where p and $1-p$ are the relative proportion of atoms with mass M_1 and M_2. The treatment is classical. It is found that the frequency of an n.m. is changed by a complex amount. The real part of it gives rise to a change of the frequency of the n.m. The imaginary part is related to a finite mean lifetime of the n.m., which, incidentally, can be measured by neutron diffraction methods [K. E. Larson et al., Ark. Fys.]. Thus the impurity scattering brings in, when the scatterers are at random, a certain amount of irreversibility, which disappears for a pure crystal. The mathematical technique used is a partial renormalization and for what remains a second order perturbation calculation (it is assumed implicitly that $|M_2-M_1| \ll M$). The explicit expression for the frequency change is given for a one-dimensional lattice and it is stated that the method can be used for higher dimensions.

The above results are used to calculate the dielectric frequency dependent tensor, which according to the method of Kubo [J. Phys. Soc. Japan **12** (1957), 570–586; MR **20** #4940a] is given by an integral over time-relaxed correlation function for the electric dipole moment. The result shows a departure of optical absorption from the Lorentz shape due to the frequency dependence of the absorption coefficient. Also an application is made to evaluate the ensemble distribution function for energy, and the results of George [Acad. Roy. Belg. Bull. Cl. Sci. (5) **45** (1959), 239–250; MR **22** #2088] are rederived in a simple way.

J. Peretti (Montpellier)

11562:

Zwanzig, Robert. Nonequilibrium processes in isotopically disordered crystals. Dependence on degree of order. J. Mathematical Phys. **2** (1961), 370–372.

The problem which is dealt with in this paper is the following. It is known that a completely ordered crystal behaves reversibly (in the harmonic approximation). It has been shown [Maradudin, Weiss and Jepsen, #11561 (MWJ)] that a completely isotopic disordered crystal has an irreversible behavior. Where does reversibility break off and be replaced by irreversibility when the amount of disorder is increased? The answer given here is that irreversible behavior occurs whenever the correlation between isotopic species has a fixed and finite range as the size of the crystal tends to infinity. If the correlation extends over a region that grows in size as the size of the crystal grows reversible behavior occurs instead.

The answer is correct to the same order of perturbation theory as the result of MWJ and the investigation is also carried on for a one-dimensional lattice. Unfortunately the result (as stated on p. 372, second section) is based on an unproved theorem. The argument presented is far from satisfactory as judged by the usual standards of mathematical rigor, but the exactness of the result seems highly probable. Further investigation is certainly needed to clear up this point.

J. Peretti (Montpellier)

11563:

Birkhoff, Garrett. Positivity and criticality. Proc. Sympos. Appl. Math., Vol. XI, pp. 116–126. American Mathematical Society, Providence, R.I., 1961.

The mathematical framework of nuclear-reactor theory consists of a set of integro-differential equations. In general, the differential operators are not self-adjoint, and the associated integral kernels are not symmetrical. This is because, physically, neutron life-histories are not reversible, even statistically. Therefore, we cannot assume the possibility of expansion into eigenfunctions; neither is it necessarily true that the eigenvalues or eigenfunctions are real, as is usually assumed in elementary treatises.

On the other hand, when reactor problems are solved on digital computing machines, one must use a "finite model" of the reactor, namely, one must subdivide the neutron phase space into a finite number of cells. The criticality problem then becomes equivalent to finding the largest eigenvalue of a finite matrix having all its off-diagonal elements non-negative. (The term "positivity" arises from this property.) The author reviews a number of theorems on such matrices, from which it follows that the largest eigenvalue is non-degenerate, and that the corresponding eigenvector (i.e., the neutron flux) is positive.

Unfortunately, the mathematics involved (Banach algebra) is quite unfamiliar to most reactor physicists, who used to refer to their mathematical ignorance by the name "physical intuition". The author thus expresses the hope that the construction of a rigorous mathematical theory of criticality will supply useful and interesting problems to mathematicians, the solution of which should be regarded with due respect by reactor physicists.

A. Peres (Haifa)

11564:

Weinberg, Alvin M. Reactor types. Proc. Sympos. Appl. Math., Vol. XI, pp. 1–19. American Mathematical Society, Providence, R.I., 1961.

The author derives the general reactor equation, both in "kernel form" and in "Boltzmann form", and discusses the approximations that can be made for various reactor types.

A. Peres (Haifa)

11565:

Kofink, W. Neue Lösungsformen der Boltzmann-Gleichung für monoenergetischen Neutronentransport in kugelförmiger Geometrie. Z. Physik **162** (1961), 489–507. (English summary)

In this paper the new solutions of the Boltzmann equation without source function for monoenergetic neutron transport in infinite medium of spherical symmetry are discussed, assuming the isotropic scattering, absorption and multiplication.

In Chapter I the reduction of the problem to the Sonine integral equation provides the singular solutions of the Boltzmann equation under consideration: The main singularity at the spherical center, being reciprocally proportional to the distance of the neutron stream from the spherical center, is stronger than another singularity at the spherical center, being proportional to $\log[(1+\mu)/(1-\mu)]$, where μ represents the cosine of the angle between the direction of the neutron and that of the radius vector. In Chapter II a couple of integral representations of the solutions, being regular at the spherical center, is discussed: While a solution concerned with the partial solution fulfills the characteristic equation, another solution expressed in terms of the half difference of two singular solutions does not need to satisfy the characteristic equation. In Chapter II the representation of both kinds of regular solutions in spherical geometry as superposition of solutions in plane geometry which belong to the discrete or to the continuous spectrum of the Boltzmann operator is provided. *S. Ueno* (Santa Monica, Calif.)

11566:

Galanin, A. D. ★Thermal reactor theory. 2nd revised and authorised edition. Translated from the Russian by J. B. Sykes. International Series of Monographs on Nuclear Energy, Division X, Vol. 3. Pergamon Press, New York-Oxford-London-Paris, 1960. xiv+412 pp. $15.00.

This is a mature exposition of thermal reactor theory, with special emphasis on heterogeneous systems. Many sections of this book are based on the work of Soviet scientists, who developed reactor theory independently of workers in other countries. An unfortunate consequence is that the notations are often different from those used in the American literature.

The book starts with a thorough discussion of diffusion, slowing down and resonance absorption, and then covers the following topics: critical size (including numerical methods), perturbation theory, fuel evolution, reactor kinetics and control, transport theory, and the general theory of heterogeneous systems.

The author has tried not only to expound the general theory of nuclear reactors, but also to examine various problems which, though slight, are often very important in practice. Much attention has been paid to the limits of accuracy and the errors of the different theoretical methods. Many formulae have been evaluated numerically so that they may be used immediately in actual calculations. Eleven appendices provide tables and graphs for day to day work. This book will therefore be found most valuable both by theoretical workers and by people engaged in the actual design of reactors.

A. Peres (Haifa)

11567:

Mohling, Franz. Very low-temperature Fermi gas. Phys. Rev. (2) **122** (1961), 1062–1090.

Using the method of the linked-pair expansion in quantum statistics introduced by him, the author develops a systematic method to solve the complicated integral equation derived by Lee and Yang, from which one can deduce the momentum distribution of a Fermi gas with strong short-range interactions at very low temperature.

It is shown that the dominant behavior of the momentum distribution at low temperature can be extracted in terms of a certain function $V'(K)$ which corresponds to the momentum distribution for the free particle system. Analyzing this function $V'(K)$, he shows that the ground-state momentum distribution has a sharp discontinuity for a low-density Fermi gas with strong short-range interactions. Also, he expresses the energy and other thermodynamic quantities in terms of $V'(K)$, so that the theory permits a generalization of perturbation theoretic results to nonzero temperatures.

He explicitly calculates the ground-state energy, momentum distribution and thermodynamic potential to third order in the scattering length and discusses a little about the contribution of fourth-order terms to the ground-state energy of the system and an application of this theory to the system of Bose gas. *R. Kubo* (Tokyo)

11568:

Mohling, Franz. Linked-pair expansions in quantum statistics. Phys. Rev. (2) **122** (1961), 1043–1061.

The author treats the Boltzmann cluster functions in quantum many-body systems, especially improvements of the method of Lee and Yang. Employing the interaction representation to obtain expressions for the cluster integrals, he introduces some new ideas in summing diagrams in order to separate the effect of particle interaction from that of overlapping wave functions due to thermal motion.

Then, following Lee and Yang, he succeeds in obtaining a convenient expansion formula of the grand potential, i.e., logarithm of the grand partition function of the system and suggests that this formulation may be useful to analyze anomalous diagrams in Brueckner-Goldstone series concerning the so-called Kohn's Paradox.

It is clear from the context that this linked-pair formulation is an extension of the contracted 0-graph formulation of Lee and Yang and also quite closely related to the method of Bloch and De Dominicis.

R. Kubo (Tokyo)

11569:

Stillinger, Frank H., Jr.; Kirkwood, John G. Quantum statistics of nonideal systems. Phys. Rev. (2) **118** (1960), 361–369.

A fugacity expansion for the grand partition function of a Bose or a Fermi gas is developed in a similar form to that given by Ursell and Mayer in the classical statistical mechanics. The grand partition function is expressed as a series of cluster integrals, each term of which is multiplied by a power of Z. Unlike the classical cluster integrals, their quantum mechanical analogs depend upon Z. The quantum analogs of the connected cluster integrals are introduced as Thiele's semi-invariants of these quantum cluster integrals in this series, and so they depend upon Z. The irreducible cluster integrals are then defined by the same relations encountered in the classical theory. Unfortunately, they cannot be regarded as virial coefficients, because they themselves depend on Z. Such a situation, however, is not limited to the present method. For example, one has a similar difficulty in C. Bloch's method, for the particle propagators appearing in this method contain an unknown chemical potential.

T. Izuyama (Tokyo)

11570:

Fujita, S.; Isihara, A.; Montroll, E. On the cluster integral development of pair distribution functions. Acad. Roy. Belg. Bull. Cl. Sci. (5) **44** (1958), 1018–1032.

The authors extend the formalism of E. W. Montroll and J. C. Ward [Phys. Fluids **1** (1958), 55–72; MR **22** #8762] to the calculation of the equilibrium pair correlation function of quantum statistical systems. The theory uses a diagram technique (statistics effects are conveniently introduced through diagrams drawn on a torus) which leads to a cluster expansion of this function in the grand canonical ensemble. The general formalism is illustrated by a derivation of two known results: the London-Placzek formula for a gas of free fermions or bosons and the Debye distribution for a classical electron gas.

R. Balescu (Brussels)

11571:

Hanks, Russell V. Cluster integral theory of distribution functions of spatially nonuniform systems. Phys. Fluids **4** (1961), 292–299.

A cluster expansion for the single-particle and two-particle equilibrium distribution functions is worked out for non-uniform systems (i.e., systems in equilibrium in an external field). The formalism is a fairly straightforward generalization of the theory of S. Fujita, A. Isihara and E. Montroll [#11570]. (The latter authors treated uniform systems.) As an application, the author derives a generalization of the London-Placzek formula for the pair distribution of non-interacting particles. He also discusses the (uniform!) plasma in a constant magnetic field.

R. Balescu (Brussels)

11572:

Zaĭcev, A. A. The use of Laplace transforms for evaluating the partition function. Ž. Èksper. Teoret. Fiz. **39** (1960), 1258–1262 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 878–880.

By using Laplace transforms, a formal evaluation of the partition function is presented, and the result is given in the form of a power series in $1/kT$.

H. Wakita (Hiroshima)

11573:

Van Hove, L.; Verboven, E. The generalized transport equation for an electron. A solution in a simple case. Physica **27** (1961), 418–432.

An application of Van Hove's earlier work [Physica **21** (1955), 517–540; **23** (1957), 441–480; **25** (1959), 268–276; MR **17**, 115; **19**, 696; **21** #3137] in which irreversibility in large quantum mechanical systems is proved with only a single, rather than repeated, assumption of the random-phase approximation. The present paper considers the case of an electron in a system of random elastic scatterers, where the perturbation is not weak. In contrast to the weak perturbation cases, where the approach to equilibrium is monotonic, here it shows damped oscillations.

G. Gioumousis (Emeryville, Calif.)

11574:

Grad, Harold. On molecular chaos and the Kirkwood superposition hypothesis. J. Chem. Phys. **33** (1960), 1342–1348.

Author's summary: "We consider properties which are possessed by typical functions of very many variables such as the n-particle molecular distribution function of a dynamical system. We show that such a function 'selected at random' will in all likelihood approximately satisfy the molecular chaos condition and, to a better approximation, will very likely satisfy the Kirkwood superposition relation."

J. Ross (Providence, R.I.)

11575:

Landsberg, P. T. ★Thermodynamics: With quantum statistical illustrations. Monographs in Statistical Physics and Thermodynamics, Vol. 2. Interscience Publishers, New York-London, 1961. x+499 pp. $14.50.

This is an outstanding modern monograph on the foundations of equilibrium thermodynamics. Although it builds up the theory from first concepts, the reader will fully appreciate the book only if he has already a good introductory knowledge of thermodynamics. From the mathematical standpoint, although the treatment is quite abstract, it is not technically difficult. The less usual mathematical concepts are reviewed in the text, so that the book should be accessible to any physicist.

The presentation of the subject (especially in the three first chapters) tends toward what the author himself calls a "geometrization of thermodynamics". From this point of view the attitude is very similar to the one taken by Carathéodory in his classical works; however, Landsberg goes much farther in this direction. He shows that the three laws of thermodynamics and their consequences can be formulated as topological properties of a certain abstract "phase space" **E**, whose coordinates are the variables needed for a complete macroscopic specification of a system in equilibrium.

The first chapter is devoted to the first law. The introduction of conceptual adiabatic and diathermic partitions enables one to define the thermal equilibrium and to introduce the empirical temperature. The author then introduces the concept of a subset β of **E** as the set of points of **E** such that between any two points of this set an adiabatic change is physically realizable and no points that can be in the set are excluded. The first law is formulated in terms of points of this set.

The second chapter, dealing with the second law, introduces first the concepts of a quasi-static process as a sequence of equilibrium states; hence, such a process is represented by a curve in the phase space **E**. If the process is moreover adiabatic, the curve lies entirely within a subset β. The author defines then an open subset γ as the interior of a subset β. The second law is formulated in terms of certain adiabatical inaccessibility assumptions within such a subset γ. This formulation generalizes and sharpens Carathéodory's well-known principle. The geometrical definition of quasi-static processes is very satisfactory from the point of view of equilibrium thermodynamics, and it is a pleasure to see how much clearer the theory becomes when one banishes the nebulous concept of "reversible processes" used in traditional expositions. This procedure excludes completely the variable "time" from the theory. This is quite satisfactory as far as equilibrium thermodynamics is considered. But one pays the price of logical elegance: the extension of thermodynamics to non-equilibrium situations becomes difficult and requires the application of "tricks" which are quite artificial. The situation can be improved only when

an appeal is made to statistical mechanics. (This is done in Chapter 5.)

The third law appears in this formulation as a certain type of unattainability principle relative to the frontiers of sets γ.

The fourth chapter is devoted to the study of general applications to various systems. Besides the development of the classical formalism, a novel feature here is the very thorough discussion of the system called "ideal quantum gas", defined thermodynamically by the relation $pv = gU$ (p: pressure, v: volume, U: internal energy, g: a constant). The most general consequences of this definition are discussed "in abstracto".

Chapter 5 deals with the combination of thermodynamics and statistical mechanics. The exposition of the principles of statistical mechanics is rather brief in the main text; just enough is given in order to fertilize the purely thermodynamical treatment. The discussion of special points of foundation of statistical mechanics is given in appendices. In this way one comes rapidly to the possibility of defining physical systems whose thermodynamics is studied thoroughly. All systems discussed in this section can be reduced to the general model of an ideal quantum gas defined above.

The general exposition is remarkable. It achieves perfectly a double goal rarely met in texts of this kind: Perfect mathematical rigor parallels extremely clear physical motivation. At each new step the logical necessity of an assumption and the physical implications of the abstract assumption are both explained in detail. Another remarkable feature of the book is its modern character. Such traditional problems as Carnot cycles are only briefly touched upon; but much stress is laid on problems of great present interest such as the theory of paramagnetism, the theory of solids, degenerate Fermi and Bose gases, information theory, etc. Even such an old problem as black-body radiation is treated in a new way and modern aspects are emphasized (e.g., radiative recombination in semiconductors).

There is a large number of suggestive and useful exercises scattered throughout the book; most of them are partially solved. This is a very good way of saving space. As a consequence, although the main emphasis is on general principles, the book is very rich in detailed results. *R. Balescu* (Brussels)

11576:

Leipnik, Roy. The extended entropy uncertainty principle. Information and Control **3** (1960), 18–25.

This paper is concerned with statistical aspects of quantum mechanics first discussed by Moyal [Proc. Cambridge Philos. Soc. **45** (1949), 99–124; MR **10**, 582] and recently developed by the author [Information and Control **2** (1959), 64–79; MR **21** #1143]. A restricted minimum principle is obtained for the entropy defined in terms of the Wigner distribution function $f(x, p; t)$ in support of the conjecture that the appropriately defined entropy of non-commuting observables has a positive minimum. *W. B. Brown* (Manchester)

11577:

Glansdorff, P. On a non-linear law of the irreversible phenomena with stationary constraints. Molecular Phys. **3** (1960), 277–283.

11578:

Shapiro, G. S. On yield surfaces for ideally plastic shells. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 414–418. SIAM, Philadelphia, Pa., 1961.

If shear forces are considered, the general shell problem involves eight stress resultants: three direct stresses, three moments, and two shears. Assuming that the shell material satisfies the Mises yield condition, the author expresses these eight resultants at yield in terms of seven parameters, thus defining the yield surface in an eight-dimensional stress resultant space. Some remarks concerning the approximation of this surface by polygonal ones are also made. *P. G. Hodge, Jr.* (Chicago, Ill.)

11579:

Naghdi, P. M. Stress-strain relations in plasticity and thermoplasticity. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 121–169. Pergamon, Oxford, 1960.

This is an excellent survey of recent developments in the field of stress-strain relations in plasticity and thermoplasticity. The first part of the survey deals with isothermal plasticity and is based on Drucker's principles, although Hill's and Prager's methods of development of the stress-strain relations are also briefly reviewed. The concepts of singular points in the loading surface, piecewise-linear loading function, and of kinematic hardening are presented in some detail. The second part of this survey deals with thermoplasticity. It starts with a brief exposition of Prager's constitutive equations for nonisothermal plastic deformation. After some basic concepts of irreversible thermodynamics, the paper ends with a review of Vakulenko's and Ziegler's contributions to the theory. A list of 80 references will be of particular value to the reader. *A. Phillips* (New Haven, Conn.)

ELASTICITY, PLASTICITY

See also 11694, 11973.

11580:

van den Dungen, F. H. Les coefficients cinematiques d'élasticité. Ann. Mat. Pura Appl. (4) **50** (1960), 291–293.

The author expresses the linear relation between stress tensor and strain tensor in the form $\tau_{ij} = \mu K_{ij}{}^{kl} \varepsilon_{kl}$, where μ is the specific mass and the coefficients K are the kinematic elastic constants. In this manner the elastic work is expressed as a function of the unit of mass instead of as a function of the unit of volume.

A. Phillips (New Haven, Conn.)

11581:

Rajagopal, E. S. The existence of interfacial couples in infinitesimal elasticity. Ann. Physik (7) **6** (1960), 192–201.

The author proposes the following axiomatic foundation for elasticity theory: "The reactions across a surface element dS in a strained elastic body consist of a force $t\,dS$ (vector part of the stress) and a couple γds (pseudo-vector stress)." The last phrase ("and . . . stress).") is the author's addition to what is otherwise equivalent to Cauchy's stress principle. Cauchy's principle is alleged to

be valid only in the absence of non-central forces between atoms. An evaluation of the stress from an atomistic model is given to substantiate this. Consequences of this extension are discussed "with a view to refute the various uncritical suggestions which have recently been made about the foundations of the theory of elasticity".

H. B. Rosenstock (Washington, D.C.)

11582:

Slezinger, I. N. The variational theorems of the non-linear theory of elasticity. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 3–4, 83–86. (Russian. English and Romanian summaries)

Author's summary: "There is shown in the paper that all known variational theorems of the non-linear theory of elasticity are particular cases of some general variational principle."

W. Urbanowski (Warsaw)

11583:

Seth, B. R. Finite deformation of plates into shells. Partial differential equations and continuum mechanics, pp. 95–105. Univ. of Wisconsin Press, Madison, Wis., 1961.

This is a summary of solutions describing bending of plates into shells within the framework of the author's slightly nonlinear elasticity theory.

J. L. Ericksen (Baltimore, Md.)

11584:

Bernstein, Barry. Relations between hypo-elasticity and elasticity. Trans. Soc. Rheology **4** (1960), 23–28.

The author classifies Truesdell's definition of hypo-elasticity and state conditions under which elastic and hypo-elastic theories define the same material. Mathematical proofs are to be published elsewhere.

A. E. Green (Newcastle-upon-Tyne)

11585:

Lakshminarayana, G. Finite strain in rotating cylinders. Appl. Phys. Quart. **6** (1959/60), 12–15.

The author considers the deformation of solid, hollow and composite circular cylinders undergoing uniform rotation on the assumption that the stresses are linear functions of the finite strains. The solution for the radial displacement is expressed as a power series in terms of the square of the angular velocity with coefficients which are functions of the radial distance from the axis. The first three terms of the series are evaluated.

J. E. Adkins (Providence, R.I.)

11586:

Seth, B. R. Stability of finite deformation. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 406–413. SIAM, Philadelphia, Pa., 1961.

Assuming an approximate linear stress-strain law, the author establishes equations of neutral equilibrium for a solid, right-circular cylinder of compressible elastic material under finite extension and torsion. Equations are solved formally, but no numerical results are obtained.

A. E. Green (Newcastle-upon-Tyne)

11587:

Reiner, M. The stress-strain-relation of elasticity and the measure of strain. Z. Angew. Math. Mech. **40** (1960), 415–420. (German and Russian summaries)

The author appears to reject his own invariant formulation of elasticity [Amer. J. Math. **70** (1948), 433–446; MR **10**, 81] as "not . . . satisfactory to the physicist". He discusses the use of various special strain measures in the hope, apparently, that a particularly simple form of stress-strain law will result. {In the reviewer's opinion, the author's paper of 1948 was on the right track while this one is on the wrong. Many solutions valid in the fully general theory have been obtained; cf., e.g., Ericksen, Z. Angew. Math. Phys. **5** (1954), 466–489 [MR **16**, 643]. The second-order effects discussed by the author have been analyzed fully and without use of any special strain measure by Rivlin [J. Rational Mech. Anal. **2** (1953), 53–81; MR **14**, 513].}

C. Truesdell (Bologna)

11588:

Pfleiderer, Hans; Seeger, Alfred; Kröner, Ekkehart. Nichtlineare Elastizitätstheorie geradliniger Versetzungen. Z. Naturforschg. **15a** (1960), 758–772.

Authors' summary: "Die Spannungsfunktionsmethode zur Lösung von Eigenspannungszuständen wird, ausgehend von der Riemann-Cartanschen Versetzungsgeometrie, in allgemeiner Form entwickelt. Die praktische Berechnung ebener Eigenspannungszustände in einem isotropen Medium in Rahmen der Elastizitätstheorie zweiter Ordnung wird im einzelnen dargestellt. Einfache Verhältnisse ergeben sich für kontinuierliche Verteilungen gerader paralleler Schrauben- oder Stufenversetzungen. Mit den dafür erhaltenen Formeln werden in quadratischer Näherung die Spannungsfelder singulärer Schrauben- und Stufenversetzungen berechnet, die in der Mittelachse hohler Kreiszylinder mit spannungsfreien Rändern liegen. Als Nebenergebnis erhalten wir die bekannte Zenersche Formel für die mittlere Volumenaufweitung bei Eigenspannungen aus der quadratischen Elastizitätstheorie."

W. Noll (Pittsburgh, Pa.)

11589:

Zahorski, Stefan. Circularly symmetric and plane shear of rubber elements in the light of the non-linear theory of elasticity. Rozprawy Inż. **8** (1960), 273–291. (Polish. Russian and English summaries)

The author bases his considerations on the form of the elastic potential

$$W = C_1(I_1-3)+C_2(I_2-3)+C_3(I_1^2-9),$$

which was proposed by himself (the notations are those of A. E. Green and W. Zerna, *Theoretical elasticity* [Clarendon, Oxford, 1954; MR **16**, 306]).

The first part is devoted to finite deformations of a cylindrical silent block bounded by an external and an internal metal tube and subject to the action of an axial force. The form of the facets is assumed to be that of an arbitrary axisymmetric surface of known equation. It is also assumed that the sum of axial stresses on this surface vanishes (a boundary condition in an integral form). The relation derived by the author between the axial force and the relative displacement of the interior surface with respect to the exterior surface is rather complicated and requires a previous solving of transcendental equations.

In the second part the author examines under the same assumptions the deformation of a plane silent block which

can be reduced to an analysis of flat shear. The deduced relation between the force and the displacement is non-linear but fairly simple.

Finally, the results are compared with those of an experiment. A better agreement is obtained than in the case of the Mooney relation ($C_2=0$).

W. Urbanowski (Warsaw)

11590:

Ramakanth, J. Dislocation of the first order. Z. Angew. Math. Mech. **40** (1960), 512–514.

Dislocations of the first order are discussed by using finite components of strain and a linear stress-strain tensor law. A convergent series solution of the corresponding non-linear differential equation is employed in place of the graphical method used by Newing and Shepherd [Philos. Mag. (7) **26** (1938), 557–569]. Numerical results are obtained, but have not been compared with known results. *B. R. Seth* (Kharagpur)

11591:

Ferrarese, Giorgio. Sulla velocità angolare nei moti rigidi e la rotazione locale nelle deformazioni finite. Rend. Mat. e Appl. (5) **18** (1959), 169–177.

Various identities for strain and local rotation, written in the homographic notations used by the Italian school, are obtained in terms of a proper vector of the rotation tensor. The conditions of integrability for this vector turn out to be the conditions of compatibility for the strain. *C. Truesdell* (Bologna)

11592:

Ferrarese, Giorgio. Le caratteristiche di deformazione complete per i solidi tubolari e una nuova forma delle condizioni di congruenza per spostamenti finiti. Rend. Mat. e Appl. (5) **18** (1959), 267–282.

Let $x^i=x^i(y^r)$ be a finite deformation, and assume that when both the x^i and the y^r are referred to the same Cartesian system, the metric tensor $C_{\alpha\beta}=\sum_i x_{,\alpha}{}^i\, x_{,\beta}{}^i$ has the form

$$\begin{Vmatrix} 1 & 0 & \gamma_2 \\ \cdot & 1 & \gamma_1 \\ \cdot & \cdot & C_3 \end{Vmatrix},$$

as is appropriate to some kinds of torsion. The author shows that γ_1 and γ_2 are necessarily linear in y^1 and y^2, respectively, while the change of volume is bilinear in y^1 and y^2. *C. Truesdell* (Bologna)

11593:

Ferrarese, Giorgio. Sulle deformazioni delle piastre. Rend. Mat. e Appl. (5) **19** (1960), 193–204.

In the notations of the preceding review [#11592], let $C_{\alpha\beta}$ be of the form

$$\begin{Vmatrix} b_1 & b_3 & 0 \\ \cdot & b_2 & 0 \\ \cdot & \cdot & 1 \end{Vmatrix},$$

as is the case in plane strain. The author shows that b_1, b_2, b_3 and the change of volume must be quadratic functions of y^3. *C. Truesdell* (Bologna)

11594:

Ferrarese, Giorgio. Sulle deformazioni finite di un solido tubolare a direttrice curvilinea. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **27** (1959), 347–355.

In suo precendente lavoro [#11592] l'autore a raggiunto alcuni risultati esatti sulle caratteristiche deformazione complete per i solidi tubolari atte a conservare le sezioni piane donando una nuova forma delle condizioni di congruenza per spostamenti finiti. Affrancandosi da ipotesi che nella configurazione di riferimento la direttrice del solido tubolare (cioè il luogo dei baricentri delle sezioni normali pensate come omogenee) è rettilinea, l'autore riprendi qui in esame lo stesso tipo di spostamento al fine di determinare le caratteristiche di deformazione complete mettersi in evidenza che in ciascuna sezione normale gli scorrimenti e il coefficiente di dilatazione cubica risultano esattamente dal quoziente di funzioni lineari delle coordinate cartesiane. Introducando coordinate curvilinee e l'omografia di deformazione l'autore deduce le condizioni di congruenza. Le più semplici deformazioni di un solido tubolare sono trattate. *D. P. Rašković* (Belgrade)

11595:

Signorini, Antonio. Questioni di elasticità. Statica non lineare; Vincoli unilaterali, statica semilinearizzata; Complementi. Confer. Sem. Mat. Univ. Bari **48–49–50**, 42 pp. (1959).

These three lectures contain remarks on some problems of elasticity according to the author's viewpoint. No mention is made of any non-Italian more recent than Kirchhoff. For example, the question of uniqueness of small deformations superposed upon large has been treated clearly and succinctly by Ericksen and Toupin [Canad. J. Math. **8** (1956), 432–436; MR **18**, 349]; the theorem of minimum energy is a special case of the thermostatics of elasticity constructed by Coleman and Noll [Arch. Rational Mech. Anal. **4** (1959), 97–128; MR **22** #413]. The author's formulation of problems involving unilateral constraints appears to be new. Most of the rest of the material appears in his numerous other publications. *C. Truesdell* (Bologna)

11596:

Bufler, Hans. Der Spannungszustand in einer geschichteten Scheibe. Z. Angew. Math. Mech. **41** (1961), 158–180. (English and Russian summaries)

Eine Anzahl von unendlich langen, homogenen und isotropen Streifen mit beliebigen Elastizitätskonstanten und beliebiger Höhe sind übereinander geschichtet. Am oberen und unteren Rand der so entstehenden Scheibe sind die Spannungen bzw. Verschiebungen vorgeschrieben. Verfasser bestimmt zunächst mittels Fouriertransformation den Spannungs- und Verschiebungszustand in einem Einzelstreifen und schreibt dann die Uebergangsbedingungen für die beiden Grenzfälle des vollkommenen Haftens bzw. des vollkommenen Gleitens (ohne Abheben) an. Für einige Sonderfälle (Halbscheibe mit aufgeklebtem Streifen, homogen geschichtete Halbscheibe bei verschwindender Reibung und elastischer Streifen auf glatter Halbscheibe) werden die Lösungen berechnet.

H. Parkus (Vienna)

11597:

Szmelter, Jan. A method of sequence of networks in

problems of elasticity. Arch. Mech. Stos. **12** (1960), 357–370. (Polish and Russian summaries)

An interesting practical method is described for finding the numerical solution of boundary-value problems of classical elasticity.

The method of sequences of networks depends on finite differences and can readily be used on high-speed computers. The rapid convergence of the method is illustrated by four numerical examples.

The method can successfully be adopted for solving other physical problems. *B. R. Seth* (Kharagpur)

11598:

Belen'kiĭ, M. Ya. Some axially symmetric problems of elasticity theory. Prikl. Mat. Meh. **24** (1960), 582–584 (Russian); translated as J. Appl. Math. Mech. **24**, 868–873.

The biharmonic stress function for axial symmetry is transformed first by a Bessel transform and then back by a Fourier cosine transform to give a new biharmonic function in the Cartesian plane, where Goursat decomposition into harmonic functions may be used. The examples given seem more easily handled by other methods. *C. E. Pearson* (Cambridge, Mass.)

11599:

Švecová, Hana. A contribution to the study of singular points in photoelasticity. Apl. Mat. **5** (1960), 401–411. (Czech. Russian and English summaries).

11600:

Rao, Kishan. Gravitational stresses on deep tunnels. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.103–C.110. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The paper is essentially an exercise based upon the reviewer's previous work bearing the same title [J. Appl. Mech. **19** (1952), 537–542; MR **14**, 515]. Calculations were carried out in a routine manner for an additional simple mapping function for the tunnel opening.

Yi-Yuan Yu (Brooklyn, N.Y.)

11601:

Mohan, Madan. A two dimensional mixed boundary value problem of elasticity for cardioid. J. Sci. Engrg. Res. **1** (1957), 203–212.

The plane-strain distribution in an elastic body, the boundary of which is a cardioid subject to mixed boundary conditions is solved by reducing the problem to the solution of a non-homogeneous Hilbert problem which determines a sectionally holomorphic function with given lines of discontinuity. A particular case is worked out in which the displacement is given by $u+iv=lz$ on part of the boundary and the rest of the boundary is free of traction. *L. M. Milne-Thomson* (Madison, Wis.)

11602:

Choudhury, P. On the effect of a certain type of nucleus of strain situated inside a semi-infinite aelotropic solid. J. Sci. Engrg. Res. **1** (1957), 225–230.

An aelotropic centre of dilatation is considered in a semi-infinite transversely isotropic body. Elliot's results [Proc. Cambridge Philos. Soc. **44** (1948), 522–533; **45** (1949), 621–630; MR **10**, 167; **11**, 289] are used to obtain the stresses in terms of Bessel's functions of order zero and one. The constants of integration are determined with the help of the Hankel inversion theorem.

B. R. Seth (Kharagpur)

11603:

Granholm, Hjalmar. Le problème de Boussinesq: une méthode de calcul approximative. Chalmers Tekn. Högsk. Handl. No. 229 (1960), 15 pp.

The author presents an approximate and physically motivated derivation of Boussinesq's classical result for the point loading of an elastic half-space, based on the division of the half-space into conical annuli.

C. E. Pearson (Cambridge, Mass.)

11604:

Rabinovič, I. Š. On a problem of equilibrium of elastic half-space. Prikl. Mat. Meh. **23** (1959), 1129 (Russian); translated as J. Appl. Math. Mech. **23**, 1613–1614.

Boussinesq's solution for a concentrated normal load applied to an elastic half-space is integrated to give the solution for a linearly-varying live load.

C. E. Pearson (Cambridge, Mass.)

11605:

Shtaerman, I. Ia. Compression of elastic bodies touching along a straight segment. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 536–545. SIAM, Philadelphia, Pa., 1961.

The author reduces the problem of elastic contact along a rectilinear segment (or curve) to solutions of coupled integral equations. *C. E. Pearson* (Cambridge, Mass.)

11606:

Aggarwala, B. D. Boundary value problems of the elastic half plane. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.41–C.66. Indian Soc. Theoret. Appl. Mech., Kharagpur.

A complex-variable decomposition of biharmonic-type functions is used to analyze generalized plane stress or bending of an orthotropic semi-infinite plate.

C. E. Pearson (Cambridge, Mass.)

11607:

Deev, V. M. On forms of the general solution of the spatial problem of the theory of elasticity with the aid of harmonic functions. Prikl. Mat. Meh. **23** (1959), 1132–1133 (Russian); translated as J. Appl. Math. Mech. **23**, 1619–1622.

11608:

Panasyuk, V. V. Development of a crack having an elliptical cross-section. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 14–19. (Ukrainian. Russian and English summaries)

The problem stated in the title is solved by reducing it to the known solution for an elastic semi-space with mixed boundary conditions. The change of the shape of the crack is discussed and by means of the minimum energy conditions the critical stresses are found. The latter generalize the Griffith critical stress. *H. Zorski* (Warsaw)

11609:

Stavraki, L. M. Principles of calculating thin-walled beams by a deformed scheme. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 143–153. (Ukrainian. Russian and English summaries)

The space problem of equilibrium of a thin-walled deformed beam is considered. It reduces to a system of four ordinary differential equations, which may be approximately regarded as linear differential equations with variable coefficients. These equations differ somewhat from the corresponding equations of V. Z. Vlasov by the presence of several additional terms.

Z. Kączkowski (Warsaw)

11610:

Payne, L. E. The elastic spindle under concentrated torques applied at the poles. J. Sci. Engrg. Res. **1** (1957), 37–42.

The paper is concerned with the axisymmetric isotropic torsion problem for a spindle, that is, a shaft of varying circular cross-section. Although the problem of a spindle subjected to two concentrated couples is of a classical nature, it seems not to have been treated before in the literature, and, in fact, the special case in which the spindle is a sphere has only recently been solved by Huber [Quart. Appl. Math. **13** (1955), 98–102; MR **16**, 767], whose result has since been obtained in a more elementary fashion by Snell [Mathematika **4** (1957), 162–165]. Introducing bipolar coordinates given by $z+ir=ic\cot\frac{1}{2}(\xi+i\eta)$ the author represents the stress function in terms of Legendre functions, obtaining expressions for the stresses in terms of elementary functions. After showing that the solution is unique, he notes that the analysis also solves the problem of a concentrated torque applied at the pole of a cavity in the shape of a half-spindle situated in an elastic half-space.

J. H. Wilkinson (London)

11611:

Deutsch, Emeric. Sur le problème de la torsion de certains cylindres élastiques isotropes. C. R. Acad. Sci. Paris **251** (1960), 2281–2283.

The torsion problem can be solved by mapping the cross-section on a unit circle. Here the author maps on a unit semicircle and applies his result to a beam of semicircular cross-section.

L. M. Milne-Thomson (Madison, Wis.)

11612:

Chakravorti, A. Note on the torsion of curved rods composed of anisotropic material with oval sections. Bull. Calcutta Math. Soc. **51** (1959), 161–166.

The problem of determing the stress distribution in an isotropic helical spring was solved by Shepherd [Quart. J. Mech. Appl. Math. **3** (1950), 459–468; MR **12**, 560]. In this note the author considers the same problem for a rod of cylindrically aeolotropic material, the cross-section of which has an oval (nearly circular) boundary. Expressions for the stress and displacements contain a parameter $n=a_{55}/a_{44}$ and reduce to Shepherd's results in the case $n=1$.

J. H. Wilkinson (London)

11613:

Frisch-Fay, R. Particular cases of large deflections. Austral. J. Appl. Sci. **11** (1960), 443–450.

The large deflection problem is solved for (a) a set of cantilevers with constant lever arm, (b) a simply-supported beam on unyielding supports and central concentrated load, and (c) a simply-supported beam with a non-symmetrical vertical load. Friction between the beam and supports is considered.

H. D. Conway (Ithaca, N.Y.)

11614:

Lehnickiĭ, S. G. Torsion of an anisotropic curved bar. Prikl. Mat. Meh. **24** (1960), 433–437 (Russian); translated as J. Appl. Math. Mech. **24**, 633–639.

It is demonstrated that Michell's theory of the twisting of circular ring sectors can be extended to the case of an anisotropic bar as long as there is elastic symmetry about plane radial cross sections.

E. Reissner (Cambridge, Mass.)

11615:

Nitsiotas, G. Die Singularitäten der frei drehbar gelagerten orthotropen Halbplattenstreifen. Ing.-Arch. **29** (1960), 223–232.

Using Pucher's method developed for isotropic plate-strips, formulae in closed form for the singularities of curvature and twist of freely supported orthotropic infinite and half-infinite strips are derived. The transverse edge in the latter case is considered to be freely supported, clamped or free. The procedure is trivially extended to rectangular plates supported along two edges. The results closely depend on the value of the parameter $\varkappa=(H/K_xK_y)^{1/2}$, where K_x and K_y are bending rigidities and H is related to torsional rigidity.

J. Nowinski (Austin, Tex.)

11616:

Tadjbakhsh, I.; Saibel, E. Large symmetrical deflections of circular plates under concentrated ring loads. Proc. Sympos. Non-Linear Phys. Probl. (Roorkee, 1959), pp. S.29–S.40. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The paper gives a solution of the von Karman differential equations for large deflection of a clamped circular plate under (i) a symmetrical single concentrated load and (ii) a symmetrical concentrated circular ring load. The loads are expressed in terms of the Dirac delta function, and the method of perturbation is applied.

C.-B. Ling (Taipei)

11617:

Olesiak, Zbigniew. Some cases of infinite isotropic plates with mixed boundary conditions. Arch. Mech. Stos. **12** (1960), 109–136. (Polish and Russian summaries)

The biharmonic equation is treated by means of Fourier transforms for several cases of mixed boundary conditions. Each problem leads to dual integral equations with solution of known form. A typical set of boundary conditions is that for $x=0$, $w=0$ for all y, M_x is a given function of y for $|y|<a$, $\partial w/\partial x=0$ for $|y|>a$.

R. C. T. Smith (Armidale)

11618:

Kerr, Arnold D. Uniformly stretched plates subjected to concentrated transverse forces. Quart. J. Mech. Appl. Math. **13** (1960), 462–471.

Assuming that the plate is governed by the equation

$\Delta\Delta w-(N/D)\Delta w=0$ where N is the intensity of tension per unit length, Green's function for an unbounded plate is used in connection with the method of images to solve plates of various shapes, simply-supported along their boundaries. Solutions are obtained for the wedge-shaped plate of angle π/m (where $m=1, 2, 3, \cdots$) and for a rectangular plate. Special cases are considered.

H. D. Conway (Ithaca, N.Y.)

11619:

Dhaliwal, R. S. Stresses in certain thin plates rotating about an axis lying in their middle plane. J. Franklin Inst. **269** (1960), 463–473.

The author applies Muskhelishvili's method to find the stresses set up by steady rotation about a line lying in the middle plane of a plate. The method is applied to contours which map on the unit circle by $z=R(\zeta+m\zeta^n)$.

L. M. Milne-Thomson (Madison, Wis.)

11620:

Ikegaya, Kazuo. Stress and deformation in conical vibrating plate. Rev. Elec. Comm. Lab. **8** (1960), 491–497.

The relationships between stress distribution, deflection and slope of a conical shell are obtained under axial loading, these apparently being of practical interest in the design of telephone diaphragms. Numerical examples are given, but studies of the vibrating plate are not made.

H. D. Conway (Ithaca, N.Y.)

11621:

Choudhury, P. On the effect of prescribed shearing stress on the plane surface of an elastic slab with varying modulus of rigidity. Bull. Calcutta Math. Soc. **51** (1959), 21–24.

Fourier integral solution for torsional deformation of an isotropic layer of finite thickness, fixed on one side and acted upon by a given rotationally symmetric distribution of shear stresses on the other. The modulus of rigidity of the material is assumed to be of the form $\mu=\mu_0 e^{kz}$, where z is the coordinate perpendicular to the plane faces of the layer.

E. Reissner (Cambridge, Mass.)

11622:

Conway, H. D. The bending, buckling, and flexural vibration of simply supported polygonal plates by point-matching. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 288–291.

11623:

Melosh, Robert J. A stiffness matrix for the analysis of thin plates in bending. J. Aerospace Sci. **28** (1961), 34–42, 64.

A matrix analysis method is presented for plates of non-uniform thickness subjected to lateral load. The method is capable of successive refinement to yield any desired accuracy. Five typical numerical examples are shown for which convergence is fairly rapid. The author recommends the method as a preliminary design approach for the elastic behavior of thin, low-aspect-ratio wings. Results presented indicate this conclusion is warranted. The method is based on the use of a stiffness matrix for an isotropic rectangular panel. The plate to be analyzed is broken up into such panels. The method should be convenient for use with computing machines.

S. Levy (Schenectady, N.Y.)

11624:

Van Fo Fi, G. A. On summation of series. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 154–160. (Ukrainian. Russian and English summaries)

Author's summary: "A method is proposed for the summation of series encountered in the theory of plates and shells. The method is effective for trigonometric series, as well as for some series constructed from special functions."

Z. Kączkowski (Warsaw)

11625:

Ambarcumyan, S. A. On the theory of bending of anisotropic plates and shallow shells. Prikl. Mat. Meh. **24** (1960), 350–360 (Russian); translated as J. Appl. Math. Mech. **24**, 500–514.

Thin elastic orthotropic plates and shallow shells of constant thickness h are considered. The author renounces the hypothesis on undeformable normals and introduces the following assumptions: (1) the distance between two points of the shell (or plate) (being before the deformation on one straight-line normal to the middle surface of the shell) after deformation remains unchanged; (2) the shear stresses $\tau_{\alpha\gamma}$ and $\tau_{\beta\gamma}$ are of the form:

$$\tau_{i\gamma}=f_i(\gamma)\varphi_i(\alpha,\beta)+\tfrac{1}{2}q_i^{+}(1+2\gamma/h)-\tfrac{1}{2}q_i^{-}(1-2\gamma/h),\quad i=\alpha,\beta,$$

where α, β are the curvilinear coordinates coinciding with the lines of principal curvature of the middle surface and with the principal directions of the elasticity, γ is the third rectilinear coordinate representing the distance along the normal from the middle surface, $f_i(\gamma)$ are functions characterizing the variation laws of shear stresses through the thickness $[f_i(\pm\tfrac{1}{2}h)=0]$, $\varphi_i(\alpha,\beta)$ are arbitrary functions of coordinates to be found, and q_i^{+}, q_i^{-} are the tangential components of the load on the both outer faces of the shell.

The equations of equilibrium give a complete system of five differential equations with respect to five unknown functions: u, v, w, φ_α and φ_β. The author shows that his method may be also used for the analysis of thick plates. Two numerical examples are given.

Z. Kączkowski (Warsaw)

11626:

Rüdiger, D. Die Verfahren von Ritz und Trefftz in der Theorie der Schalen. Z. Angew. Math. Mech. **40** (1960), 114–123.

11627:

Bieger, K. W. Die Kreiszylinderschale unter konzentrierten Belastungen. Ing.-Arch. **30** (1961), 57–62.

The author solves the three coupled differential equations of the theory of elastic cylindrical shells with respect to three components of the displacement, making use of three displacement functions. The results are expressed by a Fourier integral-series. The paper ends with two diagrams of influence surfaces of bending moments for an infinitely long cylindrical shell. *Z. Kączkowski* (Warsaw)

11628:

Lur'e, A. I. On the static geometric analogue of shell

theory. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 267–274. SIAM, Philadelphia, Pa., 1961.

The author shows that the three differential equations for the stress functions have the same form as the three differential equations of the theory of elastic shells expressed in terms of displacements. Each group of equations goes over into the other after replacement of Poisson's ratio ν by $-\nu$. Therefore the first and second boundary problems of the shell theory are concerned with the same system of differential equations and the same formulation of the boundary conditions. *Z. Kączkowski* (Warsaw)

11629:

Csonka, P. Calculation of paraboloid shells of revolution on a base comprising a regular triangle or hexagon by the relaxation method. Acta Tech. Acad. Sci. Hungar. **31** (1960), 343–357. (German, French, and Russian summaries)

In order to calculate stresses in shallow spherical membranes with equilateral triangles or hexagonal bases, the author uses the finite-difference analogue of the Poisson equation $F_{,xx}+F_{,yy}=p(x, y)$ on an equilateral triangular lattice, in conjunction with the relaxation technique. Two different procedures for the satisfaction of appropriate boundary conditions are described and a numerical example shows the practicality of the method.

E. Reissner (Cambridge, Mass.)

11630:

Bölcskei, E. Allgemeine Theorie der gekrümmten Schalen. Acta Tech. Acad. Sci. Hungar. **31** (1960), 391–423. (English, French, and Russian summaries)

A derivation of a system of thin-shell equations of linear theory, assuming isotropy, constant wall thickness and validity of the Euler-Bernoulli hypothesis. The equation of the undeformed middle surface is taken in the form $z=f(x, y)$ and the equations are established for a system of oblique surface coordinates corresponding to the Cartesian coordinates x, y in the bare plane of the shell. Appropriate pseudo-stress resultants and couples are used, together with displacement components in the directions of x, y, z. Introduction of the assumption of shallowness ($z_{,x}{}^2, z_{,y}{}^2 \ll 1$) into the equations of the paper is shown to lead to a reduction of these equations to the known shallow-shell equations.

E. Reissner (Cambridge, Mass.)

11631:

van der Neut, A. Bending at the oblique end section of cylindrical shells. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 247–269. North-Holland, Amsterdam, 1960.

The problem under consideration is the edge effect in the case of a cylindrical shell bounded by an oblique end-section. The author gives the asymptotic solution of an auxiliary problem of the cylinder bounded by an infinite helical edge under constant edge loads. He shows that this solution represents the solution for the oblique cylinder problem with an error of the order $(h/a)^{1/2}$, if the helical edge is tangent to the oblique one (here h is the wall-thickness, and a the radius of the cylinder). Applications of the theory to the angular bend of a cylindrical tube without frames at the intersection and to the bend reinforced by frames are discussed. *Z. Kączkowski* (Warsaw)

11632:

Naghdi, P. M. A note on rigid body displacements in the theory of thin elastic shells. Quart. Appl. Math. **18** (1960/61), 296–298.

It is shown that, contrary to an assertion in a foreign review article, the stress strain relations in a previous article of the author [Quart. Appl. Math. **14** (1957), 369–380; MR **18**, 839] are such that stress resultants and stress couples vanish, as they should, for the most general form of rigid-body displacements.

E. Reissner (Cambridge, Mass.)

11633:

Vainberg, D. V.; Siniavskii, A. L. Approximate analysis of shells with holes by the methods of potential theory. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 570–581. SIAM, Philadelphia, Pa., 1961.

The two-dimensional problem of the theory (due to Vlasov) of thin circular cylindrical shells is reduced to functional equations whereby the unknown functions are determined in terms of their values on the boundary. Special attention is then given to the solution for an elliptical opening using double trigonometric series.

L. S. D. Morley (Farnborough)

11634:

Koiter, W. T. (Editor). ★Proceedings of the Symposium on the Theory of Thin Elastic Shells (Delft, 24–28 August, 1959). International Union of Theoretical and Applied Mechanics. North-Holland Publishing Co., Amsterdam; Interscience Publishers, Inc., New York; 1960. ix+496 pp. $9.00.

A collection of 23 articles, all of which will be reviewed individually in these pages.

11635:

Das, A. K. Note on twisting of non-homogeneous spherical shell. Appl. Phys. Quart. **6** (1959/60), 31–32.

Author's summary: "The object of this note is to solve the problem of a non-homogeneous spherical shell, when a twisting couple acts on the outer-surface of the shell, the inner-surface being fixed."

11636:

Grigolyuk, E. I. On the unsymmetrical snapping of shells of revolution. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 112–121. North-Holland, Amsterdam, 1960.

The title problem is investigated assuming that the shell is shallow, that the pressure is uniform, and that the edge is clamped. The resulting nonlinear equations are solved by Bubnov's variational method, the approximated deflection function containing two parameters. Two nonlinear algebraic equations with respect to the deflection amplitudes are then obtained. The results for symmetrical bending may be obtained as a special case.

H. D. Conway (Ithaca, N.Y.)

11637:

Wallin, Lars. Large deflections of non-linear elastic rectangular plates. Ark. Fys. **17**, 89–95 (1960).

A rectangular membrane of incompressible (i.e., $\nu=\frac{1}{2}$)

non-linear elastic material hinged at the edges, is subjected to a uniform transverse load. The stress-strain relation is a generalization of Norton's creep law ($\varepsilon = k\sigma^n$) to three dimensions. Assuming polynomial forms for the displacements, with only three arbitrary parameters, the potential energy is minimized with the aid of a digital computer. *H. B. Keller* (New York)

11638:

Beguin, Gilbert H. A type of solution for thin shells in the form of an hyperbolic paraboloid. Z. Angew. Math. Phys. **11** (1960), 356–367. (French summary)

Consideration of solutions of the form

$$W(x, y) = w(y) \sin \alpha x, \quad F(x, y) = f(y) \cos \alpha x$$

for the differential equations,

$$D\nabla^4 W = q_z - \frac{2c}{ab}\frac{\partial^2 F}{\partial x \partial y}, \quad A\nabla^4 F = \frac{2c}{ab}\frac{\partial^2 W}{\partial x \partial y},$$

which govern the linear theory of bending of shallow shells with middle surface equation $z = cxy/ab$.

E. Reissner (Cambridge, Mass.)

11639:

Chernykh, K. F. Conjugate problems of the theory of thin shells. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 74–79. SIAM, Philadelphia, Pa., 1961.

Two boundary-value problems are said to be conjugate when either solution can be obtained from the other through an elementary interchange of field variables. Plane strain of an incompressible elastic solid is a known example, the displacement in either solution being directly related to the traction in the other. In the present paper an analogous result is obtained for homogeneous elastic shells of small uniform thickness. This is illustrated by taking the known solution for the symmetric bending of a quarter toroidal shell by edge moments.

R. Hill (Nottingham)

11640:

Seide, Paul. On the bending of circular cylindrical shells by equal and equally spaced end radial shear forces and moments. J. Appl. Mech. **28** (1961), 117–126.

The author studies the problem of how closely a set of concentrated or partially concentrated radial forces or couples (in the axial-radial plane) have to be applied around the free end of a cylinder to approximate the effect of distributing them uniformly. Using an exponential series solution of Batdorf-Donnell equations and neglecting boundary conditions at the unloaded end, he obtains curves for maximum and minimum end slopes and deflections as functions of N^2t/R, where N is the number of forces or couples and t, R are the thickness and radius. This must exceed 30 or 40 to give nearly uniform deflections under concentrated forces or couples, with greater or smaller values for other conditions.

L. H. Donnell (Ann Arbor, Mich.)

11641:

Goldberg, M. A.; Salerno, V. L.; Sadowsky, M. A. Stress distribution in a rotating spherical shell of arbitrary thickness. J. Appl. Mech. **28** (1961), 127–131.

An exact solution is given for the problem of the title, the shell being rotated about a diametral axis. Graphs are given for the maximum stress as a function of the radius ratio. Strangely enough, very few references are given.

H. D. Conway (Ithaca, N.Y.)

11642:

Černyh, K. F. Saint-Venant's problem for thin-walled tubes with a circular axis. Prikl. Mat. Meh. **4** (1960), 423–432 (Russian); translated as J. Appl. Math. Mech. **24**, 619–632.

Treating the problem as one of thin-shell theory, the author discusses the curved tube free from surface loading and carrying loads of general form at its terminal sections. Results obtained for a tube in bending are compared with those of other investigators, and with available experimental tests. *H. D. Conway* (Ithaca, N.Y.)

11643:

Mazurkiewicz, Zbigniew. The problem of bending and free vibration of a simply supported, isotropic non-homogeneous, rectangular plate. Arch. Mech. Stos. **12** (1960), 497–521. (Polish and Russian summaries)

The solution described consists in replacing the differential equation by a Fredholm integral equation of the second kind, and obtaining an infinite system of non-homogeneous linear equations in the case of equilibrium, and homogeneous linear equations in the case of vibration. It is claimed that solutions much more general than those presently known may be obtained in this manner. The article is preceded by a comprehensive summary of previous articles on the subject.

H. D. Conway (Ithaca, N.Y.)

11644:

Mitra, A. R. Note on the torsional vibration of a beam with varying rigidity. Appl. Phys. Quart. **6** (1959/60), 30.

11645:

Reid, Walter, P. Vibrating string of varying length. SIAM Rev. **1** (1959), 158–159.

Standard relationships between points on characteristics of the linear wave equation are used to derive an expression for the displacement of a string under moving end conditions. *Hirsh Cohen* (Yorktown Heights, N.Y.)

11646:

Szidarovszky, J. Natural vibration of a bar under axial force, taking the effect of shear in consideration. Acta Tech. Acad. Sci. Hungar. **31** (1960), 261–268. (German, French, and Russian summaries)

The author considers the transverse vibrations of a uniform bar, taking into account the effect of shear force but not rotary inertia. The usual techniques are applied to accomplish the solution of the pertinent fourth-order differential equation. The eigenvalues are determined explicitly for the hinged-hinged case with its particularly simple frequency equation. *W. E. Boyce* (Troy, N.Y.)

11647:

Yu, Yi-Yuan. Flexural vibrations of elastic sandwich plates. J. Aero/Space Sci. **27** (1960), 272–282, 290.

The author extends previous work on the flexural theory of elastic sandwich plates and includes the effects of transverse shear deformations, rotary and translatory inertias, flexural and extensional rigidities. With this as a goal, the author derives some of the basic properties. The shear coefficient is derived by means of simple approximations based on ratios of the density of the core and faces. For infinite plates and the usual expressions for displacement and slopes, the author derives a very complicated equation of the sixth order for frequency determination. By means of approximations of the ratio of the thickness and densities, the equation is simplified and correspondence is shown with the Timoshenko beam equation. The equations for the vibrations of an infinite plate are derived and calculations are shown comparing the various simplified equations. From his conclusions, the author shows that his main assumptions have reduced the necessary calculations without impairing the accuracy of the results of this more extensive equation. As noted for isotropic plates, the higher order of frequencies are effected by rotary inertias and shear deformations. The reviewer was rather impressed by the derivations but would wait until experimental tests be conducted to prove the validity of the author's assumptions. *H. Saunders* (Philadelphia, Pa.)

11648:

Payne, L. E. A note on inequalities for plate eigenvalues. J. Math. and Phys. **39** (1960/61), 155–159.

Let λ_1, Λ_1, and Ω_1 be the lowest eigenvalues respectively of the fixed membrane problem, the clamped plate buckling problem, and the clamped plate vibration problem on a domain D. The author shows that if D is convex, $\Lambda_1 \leq 4\lambda_1$, $\Omega_1 < \frac{1}{3}\lambda_1^2$. The first of these is sharp. The results are used to derive relations between the first two eigenvalues in buckling and in vibration of a clamped and a simply supported plate D.

H. F. Weinberger (Minneapolis, Minn.)

11649:

Oplinger, Donald W. Frequency response of a nonlinear stretched string. J. Acoust. Soc. Amer. **32** (1960), 1529–1538.

The nonlinearity is a result of the changing tension in the string as it is subjected to forced vibrations. In particular, the variable tension is assumed to be proportional to the average over the length of the string of $(\partial y/\partial x)^2$, where x and y are the usual displacements along and normal to the axis of the string. The variables in the equation of motion are separated, the time equation having solutions that are elliptic functions. The variation of the tension and the amplitude of motion are studied as functions of the frequency. *R. N. Goss* (San Diego, Calif.)

11650:

Chatterji, P. P. Vibration of a composite spherically isotropic sphere with an isotropic core. J. Sci. Engrg. Res. **1** (1957), 231–236.

Radial and rotatory vibrations of a composite sphere made up of a spherically isotropic shell with an isotropic core are considered. The solutions are obtained in terms of Bessel functions. The frequency equations are quite complicated and no numerical values are calculated.

B. R. Seth (Kharagpur)

11651:

DiMaggio, F. L.; Silbiger, A. Free extensional torsional vibrations of a prolate spheroidal shell. J. Acoust. Soc. Amer. **33** (1961), 56–58.

This short paper deals with the free torsional vibrations of a shell bounded by two confocal prolate spheroids. The differential equation is deduced from energy considerations and can be put into the form

$$[(1-\eta^2)v']' + (\lambda - c^2\eta^2 - (1-\eta^2)^{-1})v = 0.$$

Tabulated values of the solutions, the prolate spheroidal angle functions of the first kind, together with their eigenvalues, are available; the authors have prepared numerical and graphical results for the first eight modes. They also show that their solution reduces to the classical one for a spherical shell. *W. E. Boyce* (Troy, N.Y.)

11652:

Prasad, Chandrika. Transverse vibrations of a deep beam. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.123–C.128. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Assuming the plane stress conditions, the equations of motion for the transverse vibration of a thin beam are derived and a method of successive approximation is developed for the case of deep beams. It seems there are some errors in equations (19). *S. C. Das* (Madras)

11653:

Grobov, V. O. Non-stationary oscillations of a flexible shaft on elastic supports allowing for the gyroscopic effect. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 20–30. (Ukrainian. Russian and English summaries)

Der Verfasser untersucht Schwingungen eines Turborotors, also einer starren Scheibe auf nachgiebiger Welle. Die Lagerungen der Welle sind ebenfalls als nachgiebig angenommen worden mit verschiedenen Steifigkeiten in den beiden zueinander verschiedenen Richtungen. Die Kreiselkräfte der Scheibe werden berücksichtigt. Die Berechnung führt auf ein System von 4 Differentialgleichungen 2. Ordnung, in denen einige der Koeffizienten periodische Funktionen der Zeit sind. Zur näherungsweisen Bestimmung der Eigenwerte wird eine von Bogoljubov und Mitropolskii angegebene asymptotische Methode verwendet. Die Auswertung der Berechnungen zeigt, daß wegen der Verschiedenheit der Steifigkeiten der Lager kritische Drehzahlbereiche bei Frequenzen auftreten können, die der doppelten Drehzahl der Welle entsprechen. Die Kreiselkräfte der Scheibe haben dabei wesentlichen Einfluß. *K. Magnus* (Stuttgart)

11654:

Boyce, William E. Buckling of a column with random initial displacements. J. Aerospace Sci. **28** (1961), 308–312, 320.

The linearized equation of the elastic strut with freely rotating ends subject to an axial load is solved under the assumption of random initial configuration of the strut, using Green's function. The relation between the load and the mean transverse deflection at mid-height is discussed. *A. M. Freudenthal* (New York)

11655:

Weinitschke, Hubertus. On the stability problem for shallow spherical shells. J. Math. and Phys. **38** (1959/60), 209–231.

The author notes that previous analyses of the buckling under uniform pressure of shallow spherical shells, assuming rotationally symmetric deformations, have given results in disagreement with each other. His analysis of the problem, for three different kinds of support conditions, is based on power-series solutions of the two governing non-linear differential equations. A crucial feature of the method is the use of two sets of series expansions, centered at the ends of the interval of interest, with matching of the solutions at some interior point. (The author also discusses several potential variations of the technique, as well as the possible use of multiple expansion at several interior points.) The calculations were done on an IBM 704 computer, and the convergence of the series in each calculated case was confirmed by means of numerical checks of analytical convergence criteria. The calculations provide the relation between pressure and average deflection, and the buckling pressure is taken as the first local maximum of the pressure in this relation. The reliability of the numerical results thus obtained appears to be well established.

An important point underscored by this paper is that a single power-series expansion about one point may well fail to converge over the entire real interval of interest, and that such non-convergence need have no physical significance whatsoever. This is the reason that the author's results disagree with those obtained by the power-series analysis of other investigators who assumed incorrectly that the non-convergence that followed an increment of pressure necessarily implied non-existence of an adjacent equilibrium position, i.e., buckling. The author notes that experimental data is in disagreement with his theoretical results, and suggests the possibility that non-symmetrical buckling modes may be responsible; the reviewer finds this suggestion highly plausible.

B. Budiansky (Cambridge, Mass.)

11656:

Padun-Luk'yanova, L. N. Investigation of the stability of a multipanelled space framework. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 76–87. (Ukrainian. Russian and English summaries)

Making use of the method of deformation, the author gives an approximate solution of the stability problem of a space framework composed of a number of portal frames joined by two longitudinal girders.

Z. Kączkowski (Warsaw)

11657:

Chelam, E. V. Elastic behaviour of matter under very high pressures. Considerations of instability. J. Indian Inst. Sci. **42** (1960), 101–107.

The author proposes that the energetic stability criterion, usually applied to an elastic body as a whole, can also be applied locally. The reviewer agrees, provided that boundary conditions on that portion are appropriate; however, there may be overall instability without local instability.

C. E. Pearson (Cambridge, Mass.)

11658:

Barenblatt, G. I. On some basic ideas of the theory of equilibrium cracks, forming during brittle fracture. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 21–38. SIAM, Philadelphia, Pa., 1961.

This is a survey of the author's approach to equilibrium of elastic bodies containing cracks, i.e., surfaces of discontinuity of the displacements. The author follows S. A. Hristianovič's idea [Želtov and Hristianovič, Izv. Akad. Nauk SSSR Otd. Tehn. Nauk. **1955**, no. 5, 3–41] that the opposite sides of a crack come very close to each other in the end region of the crack and involve there an action of intensive molecular cohesion forces. Two hypotheses are introduced, namely, the crack's end region is very small compared with the entire crack and the shape of the end region's cross section is always the same. A theorem is derived that the actual configuration of a system of cracks corresponds to an extreme value of the elastic potential. The state of stress in the neighbourhood of a boundary point of the crack is analyzed by means of a method of complex variables function [N. I. Muskhelishvili, *Some basic problems of the mathematical theory of elasticity*, Noordhoff, Groningen, 1953; MR **15**, 370]. The result of the analysis is of quantitative character: the rupture stress at the contour of the crack is finite, and the opposite sides of the crack close smoothly on its contour. If the cohesion forces are neglected the normal stress on the contour of the crack will be infinite according to the law $Y_y = K/\pi\sqrt{s}$, where s is the distance and K is a constant characteristic for the material. In the author's opinion, a satisfactory picture of the state of stress of a cracked solid may be obtained by an elastic solution taking into account the above singularity. The "modulus of cohesion" K and density of surface energy T, which is the basic constant in energetic consideration, are related by $K^2 = \pi ET/(1-\nu^2)$.

J. Murzewski (Kraków)

11659:

Budiansky, B. Buckling of clamped shallow spherical shells. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 64–94. North-Holland, Amsterdam, 1960.

The non-linear boundary-value problem for rotationally symmetric deformations is converted into a pair of integral equations. This system contains a loading parameter p, a geometrical parameter λ; and a new average deflection parameter ρ is introduced. For fixed λ, iterative solutions are sought numerically by adjusting either p or ρ. Critical pressures are defined as local maxima of $p(\rho)$ or $\rho(p)$. Such points are either accurately determined or defined by a "spectacular" loss of convergence in the calculations. By including small rotationally symmetric initial imperfections in the calculations it is shown that they do not significantly alter the form of $p_{cr}(\lambda)$ found for the perfect cap. Relaxing the clamped edge condition yields a related problem whose "buckling loads" are determined exactly as the eigenvalues of the corresponding linear problem. Some heuristic arguments are presented in order to relate qualitatively the computed critical loads to these exact values.

The author compares his $p_{cr}(\lambda)$ with the experimentally determined buckling loads of actual caps and "critical pressures" determined by other investigators who frequently use different definitions of p_{cr}. These in general do not agree for "larger" values of λ. It is concluded that non-axisymmetric theories are required to explain the experiments. However, the reviewer and E. Reiss [J. Aero/Space Sci. **26** (1959), 643–652; MR **21** #6808] have obtained

good agreement with experiments for a limited range of λ from a symmetric theory using a different definition of buckling load. *H. B. Keller* (New York)

11660:

Mettler, E.; Weidenhammer, F. Kinetisches Durchschlagen des schwach gekrümmten Stabes. Ing.-Arch. **29** (1960), 301–315.

In der Statik ist das Durchschlagen des gekrümmten Stabes ein wohlbekanntes Stabilitätsproblem. Ein ganz entsprechendes kinetisches Stabilitätsproblem entsteht, wenn der gekrümmte Stab unter der Einwirkung einer schwingenden Querbelastung steht. Der Autor betracht einen an beiden Enden in festen Gelenken gelagerten Stab mit der kleinen spannunglosen und zeitlich konstanten Vorausbiegung und der zusätzlichen, von der Zeit abhängingen kinetischen Querverschiebung. Der Stab trage die orts- und zeitabhängige Querlast je Längeeinheit. Die Bewegungsgleichungen und Endbedingungen sind aus dem Prinzip von Hamilton hergeleitet. Die Formänderungsenergie wurde nach Marguerre und Kappus in einer Näherung berechnet. Zur näherungsweisen Lösung verwendet man die Annahmen der technischen Biegelehre und die Kirchhoffsche Annahme und eine inhomogene nichtlineare Integrodifferentialgleichung für die Biegeschwingung aufgestellt ist. Ein Lösungsansatz mit zeitabhängigen Funktionen führt diese partielle Integrodifferentialgleichung für eine beliebige Lastverteilung auf ein System nichtlinearer gewöhnlicher Differentialgleichungen zurück. Durch diesem System werden weiterhin die erzwungene Schwingungen beschrieben; dabei werden nur die erste vier Gleichunger. verwendet. Mit Hilfe von Störungsrechnung sind diese Schwingungen berechnet. Es wird gezeigt, dass diese Schwingungen wesentlich den gleichen Frequenz-Amplituden-Zusammenhang wie bei Duffingschen Schwingers haben. Das Stabilitätsverhalten dieser Schwingungen liess sich mit Hilfe der Methode der langsam veränderlichen Phase und Amplitude näherungsweise ebenfalls aus den entsprechenden Eigenschaften des Duffingschen Schwingers verstehen.

D. P. Rašković (Belgrade)

11661:

Chudzikiewicz, Andrzej. Stability loss due to the deformation of the cross-section. Rozprawy Inż. **8** (1960), 45–61. (Polish. Russian and English summaries)

The author investigates the loss of stability in axial compression of a thin-walled bar with a closed square cross-section; with the possibility of a deformation of the cross-section and the axis of the bar remaining rectilinear. The walls buckle in their planes. The end-sections are assumed to be stiffened by ribs and are free to warp. One of the walls, compressed with a force $P/4$ and loaded by the two reactions r resulting from the interactions of the two neighbouring walls are taken as plates subject to deformation and are calculated from the equation of a plate compressed in one direction. The critical load results from a transcendental equation, and may be expressed in terms of the Eulerian buckling force by using the coefficient χ, which may be computed from a transcendental equation containing non-dimensional terms only. Here the coefficient χ is defined by $P_{cr}=\chi \cdot P_e$.

The exact proportion of dimensions for $\chi=1$ has been computed and represented in Table 1. When $\chi=1$, there is no interaction between the walls, which then buckle independently. Next, the author deduces an approximate solution by means of the energy method, assuming the deflection surface to be of double-sine form: $w(x, y)=A \sin(n\pi x/l) \sin(m\pi y/l)$ $(n=1, 2, 3, \cdots; m=1, 3, 5, \cdots)$. The values of χ so obtained differ by no more than 1% from the exact solutions. Finally, a transcendental equation of stability is derived for the plastic range on the basis of the shear modulus hypothesis and of the Bleich theory.

According to the author's theory, the critical loads may be considerably lower than the Eulerian buckling force, but that for the usual dimensions of steel bars, this case is unlikely to occur, either in the elastic or in the plastic range. On the contrary, in bars made of materials with lower modulus of elasticity and high elastic limit, the buckling form considered may prove to be important practically. *J. S. Nałeszkiewicz* (Warsaw)

11662a:

Gilvarry, J. J. Fracture of brittle solids. I. Distribution function for fragment size in single fracture (Theoretical). J. Appl. Phys. **32** (1961), 391–399.

11662b:

Gilvarry, J. J. ; Bergstrom, B. H. Fracture of brittle solids. II. Distribution function for fragment size in single fracture (Experimental). J. Appl. Phys. **32** (1961), 400–410.

Under the assumption that fragmentation proceeds by "stress-activation" of volume, face, and edge flaws, the distributions of which are independent of each other, it is shown that the density function of fragment size $p(l, s, v)$, defined in terms of edge length l, face area s and volume v, can be represented by a Poisson distribution involving a linear combination of l, s and v, provided only first fractures are considered.

The theoretical results obtained in Part I are checked by fracturing spherical glass specimens under pressure. The predicted linear variation with the mean-particle dimension is confirmed and the inapplicability to first (non-repeated) fracture of the logarithmic-normal distribution of fragments is shown.

A. M. Freudenthal (New York)

11663:

Wah, Thein; Gregory, R. Kirk. Creep collapse of long cylindrical shells under high temperature and external pressure. J. Aerospace Sci. **28** (1961), 177–188, 208.

Authors' summary: "This paper describes theory and tests of the creep collapse of long thin aluminum-alloy cylinders under external radial pressure. Steady-state creep is assumed in the theoretical derivation. The test temperatures were between 300° and 500° F. The collapse time for each cylinder was calculated theoretically. Agreement between theoretical and test results was fair."

11664:

Miles, J. W. The propagation of an impulse into a viscous-locking medium. J. Appl. Mech. **28** (1961), 21–24.

The author considers one-dimensional propagation through a viscous-locking medium, which is defined to

behave like a viscous liquid until a critical compressive strain is reached, and thereafter remain rigid. By analogy with a freezing problem in heat-conduction theory, a similarity solution is obtained for impulsive loading of the boundary, when an expanding rigid region advances into the fluid. It is found that the peak pressure may lie either at the rigid-fluid boundary or within the fluid region, but in both cases falls off inversely as the square of the distance. In the basic momentum balance equation, the author assumes that the total momentum in the fluid region remains constant once the medium boundary becomes rigid, not allowing momentum flux across the rigid-fluid boundary. Although this is true for the above similarity solution, there appears no reason why it should be valid in general. A solution for arbitrary loading is given for the more simple ideal-locking medium, which supports no pressure until becoming rigid at the critical strain.

L. W. Morland (Sevenoaks)

11665:

Craggs, J. W. On two-dimensional waves in an elastic half-space. Proc. Cambridge Philos. Soc. **56** (1960), 269–285.

Assuming that the stress components depend on r/t and θ (r, θ polar coordinates), the author uses the method of dynamic similarity to solve some problems of dynamic stress propagation in a two-dimensional elastic half space. The basic problems solved are those of constant stress on $\theta=0$, $0<r/t<v$, where v is a constant and concentrated load at the origin. Using these basic solutions many other problems may be solved.

G. Eason (Newcastle-upon-Tyne)

11666:

Musgrave, M. J. P. Reflexion and refraction of plane elastic waves at a plane boundary between aeolotropic media. Geophys. J. **3** (1960), 406–418.

The author presents a qualitative discussion of the general title problem along with numerical results for the reflection of body waves at the free surface of some hexagonal media. For particular representative materials (ice, beryl, zinc) results are given for slownesses, amplitudes, and energy fluxes of body waves generated through the incidence of a body wave on differently oriented boundaries. The author shows when the material is only slightly anisotropic, like ice and beryl, the assumption of an equivalent isotropic medium may be a good approximation to the reflection characteristics of these materials, subject to certain limitations. The reflection characteristics of a material like zinc, on the other hand, vary to such a degree with boundary orientation, that an isotropic approximation is not very useful.

J. Miklowitz (Pasadena, Calif.)

11667:

Toupin, R. A.; Bernstein, B. Sound waves in deformed perfectly elastic materials. Acoustoelastic effect. J. Acoust. Soc. Amer. **33** (1961), 216–225.

This is a concise, general treatment of the propagation of infinitesimal plane waves in undeformed and homogeneously deformed perfectly elastic isotropic and anisotropic materials. Diverse results discussed include necessary and sufficient conditions that linear elasticity theory describe such waves for a given material and an analysis of the acousto-elastic effect in isotropic materials.

J. L. Ericksen (Baltimore, Md.)

11668:

Geertsma, J.; Smit, D. C. Some aspects of elastic wave propagation in fluid-saturated porous solids. Geophysics **26** (1961), 169–181.

Biot's equations are used to analyze wave propagation and attenuation in fluid-saturated porous elastic solids without and with interfaces.

A. M. Freudenthal (New York)

11669:

Gilbert, Freeman. Scattering of impulsive elastic waves by a smooth convex cylinder. J. Acoust. Soc. Amer. **32** (1960), 841–857.

The purpose of this article is to obtain the short-time approximations for the scattering of impulsive elastic waves by a smooth convex cylinder. In the first part of the article, solutions are given for the scattering of elastic waves by a circular cylinder when the exciting field is a line source. The results are expressed in integral form and then evaluated asymptotically to yield the first motion (short time) approximations. In the second part of the article, Keller's geometrical theory of diffraction [*Calculus of variations and its applications*, Proc. Symp. Appl. Math., Vol. 8, pp. 27–52, McGraw-Hill, New York, for Amer. Math. Soc., 1958; MR **20** #640] is combined with the first motion approximations to solve the problem of scattering by a smooth convex cylinder. The results are valid for pulses whose durations are short compared to the travel time across the cylinder.

F. C. Karal (New York)

11670:

Kalnins, A.; Naghdi, P. M. Propagation of axisymmetric waves in an unlimited elastic shell. J. Appl. Mech. **27** (1960), 690–695.

Using an exact system of linearized equations of motion for the thin shallow spherical elastic shell (derived earlier by the second author), the authors treat the problem of the unlimited shell subjected to an oscillatory normal point load at the apex. The resultant steady-state-traveling wave solution for the problem is analyzed through approximations, reduction to special cases, and numerical evaluation. In particular, curves are given exhibiting the wave form of the axial displacement (of the shell middle surface) as a function of distance from the apex, for various fixed forcing frequencies and time values, for a typical thin shallow shell.

When the longitudinal inertia is neglected, the present theory reduces to that given by Reissner for transverse vibrations of shallow shells [Quart. Appl. Math. **13** (1955), 169–176; MR **16**, 1070]. The authors point out that the analogous solution obtained by Reissner on the basis of his theory [ibid., 279–290; MR **19**, 905] has standing wave character for the low frequencies, whereas their solution has traveling wave character for all frequencies. For high frequencies the two solutions give similar results.

J. Miklowitz (Pasadena, Calif.)

11671:

Gol'dberg, Z. A. Interaction of plane longitudinal and transverse elastic waves. Akust. Ž. **6** (1960), 307–310 (Russian); translated as Soviet Physics. Acoust. **6** (1961), 306–310.

The author's work is concerned with the interaction of plane body waves occurring in an infinite isotropic elastic

solid governed by equations of motion that account for the square terms in the strain tensor. The work points out that a transverse wave cannot exist without a longitudinal wave, whereas the latter can propagate without the former. Further, it is shown that the shape of a transverse wave does not vary during propagation, but that of a longitudinal wave does.

J. Miklowitz (Pasadena, Calif.)

11672:

Hangan, M. Sur la distribution des efforts intérieurs pendant les surcharges des pièses sollicitées par une compression excentrique. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 3–4, 313–322. (Russian and Romanian summaries)

This paper deals with the elementary theory of bending with axial force of a beam when the stress-strain law is nonlinear. The author expresses the stress as a power series of the strain and observes that when the stress-strain law is nonlinear the stress will increase with the external load less than proportionally over a portion of the cross-section and more than proportionally over the remainder of the cross-section. The curve separating these two portions is discussed.

A. Phillips (New Haven, Conn.)

11673:

Sesterikov, S. A. On the construction of a theory for an ideally plastic body. Prikl. Mat. Meh. **24** (1960), 412–415 (Russian); translated as J. Appl. Math. Mech. **24**, 604–608.

The question considered is the proper form of the yield condition for an isotropic perfectly plastic material. For such a material convexity, isotropy, incompressibility and a knowledge of the yield point in a single direction determine two bounding prisms for the yield surface in principle stress space. According to the author, D. D. Ivlev [Prikl. Mat. Meh. **22** (1958), 850–855; MR **21** #6146] showed that among all possible yield surfaces the Tresca condition minimized the work done by the stresses on given incremental strains.

The author claims that Ivlev's result is fortuitous and depends upon choosing a state of simple tension as a reference point. The Mises condition, on the other hand, gives a value for the incremental work which is independent of the reference stress point. Since this property is invariant it is a more suitable criterion for choosing a yield condition, according to the author.

The reviewer is not impressed by either argument. Experimental evidence indicates that neither the Mises or Tresca condition is precisely accurate and that either represents a rather close approximation. Abstract reasoning as to which of the two has a better philosophical basis does not seem to throw any light on the subject.

P. G. Hodge, Jr. (Chicago, Ill.)

11674:

Colonnetti, Gustavo. The interpretation of plastic deformations as heredity phenomena. Problem of continuum mechanics (Muskhelishvili anniversary volume), pp. 80–92. SIAM, Philadelphia, Pa., 1961.

Non-hereditary phenomena are defined as those where the future of a system is fully defined by the present state (including time derivatives); other phenomena where the future depends on previous experience are called hereditary. Non-hereditary phenomena lead generally to differential equations for their solution; hereditary phenomena to integro-differential equations.

Following early work by V. Volterra, the author expands the hereditary functions in a series analogous to a Taylor series. He then terminates these series with one or two terms and assumes simple forms for the various defining functions. With the resulting simplified theory he demonstrates qualitative agreement with experimental results in simple inelastic tests for steel and reinforced concrete.

P. G. Hodge, Jr. (Chicago, Ill.)

11675:

Murch, S. A. On the pure bending of an elastic, perfectly plastic curved bar in the state of plane strain. J. Franklin Inst. **270** (1960), 301–316.

The problem of the title is analysed for a material compressible within elastic regions and obeying the Prandtl-Reuss stress-strain relations. The solution of the same problem due to Shaffer and House [J. Appl. Mech. **22** (1955), 305–310] is found to be valid for compressible material only within the limited range of the bar radii ratio. The present paper furnishes the remaining stress profiles and thus gives the complete stress pattern of the elastic-plastic problem. The associated displacement field is also found.

A. Sawczuk (Warsaw)

11676:

Sokolovskii, V. V. A problem of plasticity theory. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 513–518. SIAM, Philadelphia, Pa., 1961.

The problem is that of drawing a strip of rigid/plastic material between a pair of wedge-shaped dies under conditions of plane strain. For the yield condition a special (strong) dependence of maximum shear stress on mean pressure is assumed such that, over a limited range of pressure, the field equations for stress are hyperbolic with straight characteristics. This allows a closed solution for the drawing problem over a certain interval of thickness reductions and die angles. No associated velocity field or uniqueness theorem is given; indeed a flow rule is not even mentioned. For the present, therefore, the status of this ingenious solution is undecided.

R. Hill (Nottingham)

11677:

Hodge, Philip G., Jr. A comparison of yield conditions in the theory of plastic shells. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 165–177. SIAM, Philadelphia, Pa., 1961.

Exact yield conditions for shells of Tresca or Mises rigid/plastic material are compared with two approximating linear relations. The latter are then used in the extremum principles of rigid/plastic theory to obtain close estimates of the yield-point interaction curve for an annular plate under combined tension and bending.

R. Hill (Nottingham)

11678:

Phillips, Aris. Pointed vertices in plasticity. Plasticity: Proceedings of the Second Symposium on Naval

Structural Mechanics, pp. 202–214. Pergamon, Oxford, 1960.

All flow-law theories of plasticity are based on the assumption that there exists a "yield surface" in a suitably chosen stress space and that plastic flow can take place only when the stress point is on the surface. For a strain-hardening material the yield surface will change with stress history. According to some theories, such as isotropic or kinematic hardening, the yield surface essentially retains its initial shape and hence will be smooth or not depending upon its initial properties. However, according to so-called slip theory, the yield surface will change its shape in such a way that the current stress point is always at a point of discontinuous slope, i.e., a pointed vertex.

As the author points out, experiments which must consider finite increments of stress and strain cannot distinguish between a true pointed vertex and a region of very large but finite curvature. Within this limitation, several experimental studies by other authors are quoted, some of which did, and others of which did not, find pointed vertices. He then quotes several of his own experiments which indicate the existence of such vertices.

P. G. Hodge, Jr. (Chicago, Ill.)

11679:

Cowper, G. R. The elastoplastic thick-walled sphere subjected to a radial temperature gradient. J. Appl. Mech. **27** (1960), 496–500.

A thick-walled sphere of an elastic, perfectly plastic material is subjected to a monotonically increasing temperature differential t on its two faces. A complete solution is found for any sphere of radius ratio less than 2.791. As t is increased, the sphere is first entirely elastic. A plastic region next forms at the inner surface and later a second plastic region forms at the outer surface. The width e of the central elastic region tends asymptotically to zero as t tends to infinity.

The complete solution can be expressed as a power series in the parameter e. The author then shows that an approximate solution for large t can be easily obtained by neglect of the elastic strain components, and that this approximate solution is, in fact, the leading term of the asymptotic series for the exact solution. Further, if the approximate stress solution is substituted back into the exact (including elastic effects) flow law, an improved approximation, correct to two terms of the series, is obtained for the displacements.

P. G. Hodge, Jr. (Chicago, Ill.)

11680:

Sankaranarayanan, R. Plastic interaction curves for circular cylindrical shells under combined lateral and axial pressures. J. Franklin Inst. **270** (1960), 359–366.

A circular cylindrical shell, clamped at one end and free at the other, is subjected to external radial and axial pressures P and Q. Under the assumption that the shell may be approximated by an ideal sandwich shell of a rigid, perfectly plastic material, one quadrant of the interaction curve defining the plastic yield point in a (P, Q)-plane is obtained. The associated stress and velocity fields define a complete solution for all values of the shell-geometry parameters. *P. G. Hodge, Jr.* (Chicago, Ill.)

11681:

Kuznecov, A. I. The problem of non-homogeneous plastic layer. Arch. Mech. Stos. **12** (1960), 163–172. (Russian. Polish and English summaries)

Plastic flow of a plastically non-homogeneous layer subjected to compression between two rough plates is studied in the paper. The classical solution by L. Prandtl is reproduced for a material characterised by the yield point varying linearly across the sheet thickness. For compression of a plate of finite dimension it is found that the analogy between the surface pressure distribution and the shape of a sand-hill [cf. A. A. Il'yušin, Prikl. Math. Meh. **18** (1954), 265–288] is not valid if the yield limit is space variable. An analogy between the considered problem and the plastic torsion of a plastically non-homogeneous bar is formulated.

An example of rectangular sheet is worked out for a particular form of the yield point function. Characteristics are obtained numerically. *A. Sawczuk* (Warsaw)

11682:

Heyman, Jacques. Progress in plastic design. Plasticity: Proceedings of the Second Symposium on Naval Structural Mechanics, pp. 511–537. Pergamon, Oxford, 1960.

This survey paper deals with some recent progress in plastic design. Some of the topics discussed are: reinforced concrete, tall buildings column design, and digital computers. *A. Phillips* (New Haven, Conn.)

11683:

Payne, H.; Czyzak, S. J. On the torsion of a thin-wall cylinder following a plastic extension. J. Mech. Phys. Solids **8** (1960), 39–44.

The classical model of Taylor for the derivation of the stress-strain relation for a crystalline aggregate in the plastic region has recently been extended by Payne [same J. **7** (1959), 126–134] to include nonlinear strain paths. The authors apply this modified theory to the problem of a thin-walled cylinder of a face-centered cubic polycrystalline material subjected to an incremental torsional strain on an existing tensile or compressive plastic strain. They compare their theoretical results with available experimental results. *A. Phillips* (New Haven, Conn.)

11684:

Eason, George. The minimum weight design of circular sandwich plates. Z. Angew. Math. Phys. **11** (1960), 368–375. (French summary)

The systematic method of treatment of the general problem of the minimum weight design of perfectly-plastic, rigid structures, due to D. C. Drucker and R. T. Shield [see Quart. Appl. Math. **15** (1957) 269–281; MR **19**, 970], is here applied to so-called sandwich plates. Circular symmetry is imposed, and attention is given to von Mises material. Solutions of the stated problem are given for cases when there is a circular or an annular distribution of applied load. It is found that the solutions appertaining to von Mises and Tresca materials are the same when the plate edge is simply-supported; but the analysis pertaining to von Mises material is considerably more complicated than that pertaining to Tresca material when the

plate edge is built-in, although no significant differences apparently exist between the final numerical results relating to these materials. *H. G. Hopkins* (Sevenoaks)

11685:

Safrončik, A. I. Rotation of a cylinder with a variable angular velocity in a visco-plastic medium. Prikl. Mat. Meh. **23** (1959), 1051–1056 (Russian); translated as J. Appl. Math. Mech. **23**, 1504–1511.

The author examines the two-dimensional flow of a linear visco-plastic material, infinite in extent, which is set in motion by the rotation of a rigid circular cylinder. An equation is then found for the determination of the radius of propagation of the flow. A brief discussion is given of the corresponding problem for a bounded medium.

J. E. Adkins (Providence, R.I.)

11686:

Faupel, J. H. Some considerations of the mechanics and design limitations of autofrettage. J. Franklin Inst. **269** (1960), 474–489.

Autofrettage is a process in which a thick-walled cylinder is loaded into the partially plastic range by internal pressure for the purpose of producing favorable residual stresses. The author quotes various theoretical and experimental results in a qualitative discussion of some of the factors to be considered in practical applications of autofrettage. *P. G. Hodge, Jr.* (Chicago, Ill.)

11687:

Rozenblyum, V. I. Plasticity conditions for thin shells. Prikl. Mat. Meh. **24** (1960), 364–366 (Russian); translated as J. Appl. Math. Mech. **24**, 520–524.

For a body in an essentially plane state of stress the Mises yield criterion is

$$\text{(A)} \qquad P_t^2 \equiv t_1^2 - t_1 t_2 + t_2^2 + 3t_{12}^2 = 1,$$

where the t's are dimensionless direct stress resultants. For a similar body in a state of pure bending, the yield condition is $P_m^2 = 1$, where P_m is defined by an expression similar to (A) involving dimensionless moment resultants. The author shows that, for a thin shell in an arbitrary state, the projection of yield criterion in a (P_t, P_m)-plane must be bounded from above by the square $P_t^2 = 1$, $P_m^2 = 1$ and from below by the square $|P_t| + |P_m| = 1$. He suggests the curve $P_t^2 + P_m^2 = 1$ as a convenient approximation intermediate to these two bounds.

P. G. Hodge, Jr. (Chicago, Ill.)

11688:

Favre, Henry. Sur une loi de seconde approximation régissant la déformation d'un sol horizontal sous l'action d'une charge répartie sur un segment de droite. C. R. Acad. Sci. Paris **251** (1960), 2653–2655.

11689:

Chang, Chieh-Chien; Ebcioglu, Ibrahim K. Thermoelastic behavior of a simply supported sandwich panel under large temperature gradient and edge compression. J. Aerospace Sci. **28** (1961), 480–492.

The title problem is investigated by first formulating the equations through the use of the principle of minimum potential energy. Some preliminary theoretical and experimental results are shown. The limiting case of an infinitely wide sandwich panel under cylindrical bending is further treated. *Yi-Yuan Yu* (Brooklyn, N.Y.)

11690:

Sneddon, I. N.; Tait, R. J. On Lur'e's solution of the equations of thermoelastic equilibrium. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 497–512. SIAM, Philadelphia, Pa., 1961.

Lur'e has presented a solution to the field equations of linear thermoelasticity for steady-state temperature distributions which is suitable for problems concerning semi-infinite solids and thick plates with traction-free boundaries [A. I. Lur'e, *Prostranstvennye zadači teorii uprugosti*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1955; MR **17**, 800]. This solution reduces the problem to the determination of a harmonic function ψ and a function $\chi(z)$ quadratic in z, where z is the coordinate perpendicular to the plane boundaries.

In this paper this method is exploited in a semi-inverse fashion by specification of the functions ψ and χ to obtain solutions of problems of physical interest, a number of which had been obtained previously by other means. Problems for which solutions are presented include that of the semi-infinite solid and the thick plate, and an elastic plug in the form of a rectangular parallelepiped or a circular cylinder of finite height contained in a rigid, perfectly-insulating plate with thickness equal to the height of the plug. Integral-transform techniques, double Fourier series and Dini series are utilized, as appropriate, in the specification of ψ to obtain solutions corresponding to general temperature distributions on the traction-free boundaries. Numerical evaluations of the results are promised in a future publication.

J. H. Weiner (New York)

11691:

Boley, B. A. Thermal stresses. Structural mechanics, pp. 378–406. Pergamon Press, New York, 1960.

A brief survey of thermal stress problems is given. The author starts from the coupled heat equation (effect of linear viscoelasticity is also taken into account) and discusses the conditions of neglecting the coupling terms. The effect of inertia in uncoupled problems is investigated for simple one-dimensional cases of suddenly heated beams and plates. Next, the author presents the variational formulation of heat-conduction theory due to Biot. The concepts of thermal potential and the dissipation function are introduced. A practical aspect such as the thermal conduction of structural joints, melting of bodies under conditions of high surface heating, are discussed. Next, a survey of exact thermoelastic solutions is given. Certain relations between the theory of generalized plane-stress and the three-dimensional theory are examined. The following part of the survey is devoted to thermoelastic strength-of-material theory, thermoelastic Saint-Venant principle and some general results which are derived on the basis of the extension of isothermal Betti's method to include thermal effects. The two last parts of the survey concern a few non-linear problems of thermoelasticity (thermoelastic stability, viscoelasticity and thermal plastic deformations). *Witold Nowacki* (Warsaw)

11692:

Piechocki, Władysław. A quasi-steady-state thermoelastic problem of a circular disc. Rozprawy Inż. **8** (1960), 93–100. (Polish. Russian and English summaries)

The subject of this report is the determination of the state of displacement and stress in a circular disc due to the action of heat sources of intensity W. Isotropy and homogeneity are assumed both for thermal and elastic properties. The quantities characterizing the material are assumed to be constant and independent of temperature. It is assumed that the heat sources vary harmonically in time. The problem is treated as quasi-steady-state, that is, the inertia terms in the equations of elasticity are rejected. Free and clamped edge conditions are considered.

Witold Nowacki (Warsaw)

STRUCTURE OF MATTER

See also 11529, 11683, 11839, 11845, 11846.

11693:

Lyons, D. H.; Kaplan, T. A. Method for determining ground-state spin configurations. Phys. Rev. (2) **120** (1960), 1580–1585.

The mathematical problem involved here is the minimization of a quadratic form $E=\sum_{n\nu,m\mu,t} J_{n\nu,m\mu} S_{n\nu,t} S_{m\mu,t}$ subject to "strong" constraints $\sum_t S_{n\nu,t} S_{n\nu,t}=S_\nu^2$. Instead of solving this problem directly, the authors consider the problem obtained by replacing the strong constraints by one "weak" constraint, the latter derived by multiplying each strong constraint by a positive number $\beta_{n\nu}$ and summing over all n and ν. The weak constraint is thus not a sufficient condition for validity of the strong ones, but the solution to the modified problem is shown to satisfy the original problem as well in certain cases, provided the $\beta_{n\nu}$ are suitably chosen. [The case $\beta_{n\nu}=1$ has been considered earlier by J. M. Luttinger and L. Tisza, Phys. Rev. (2) **70** (1946), 954–964.] The following physical situations are considered in detail: linear chain with two types of spin, Néel configuration in cubic spinels, and Yafet-Kittel configuration in non-cubic spinels.

H. B. Rosenstock (Washington, D.C.)

11694:

Rajagopal, E. S. The rotational invariance condition in lattice dynamics. Ann. Physik (7) **6** (1960), 182–191.

The rotational invariance condition derived in some general treatments of lattice dynamics [M. Born and Kun Huang, *Dynamical theory of crystal lattices*, Clarendon, Oxford, 1954; p. 222] is alleged to be incorrect on account of having been derived from an expression for the displacements which does not correspond precisely to a rigid-body rotation. This result is alleged to have "considerable effect" on several applications, such as photoelasticity and piezoelectricity. *H. B. Rosenstock* (Washington, D.C.)

11695:

Ramachandran, G. N. Analysis of the X-ray diffraction pattern of helical structures. Proc. Indian Acad. Sci. Sect. A **52** (1960), 240–254.

The theory of the diffraction pattern of helical structures having an integral or rational number n of units per turn is here generalized to the case that n is arbitrary. Instead of using repeat spacing along the axis of the helix as the standard of reference (as had been done previously, but which is meaningless if n is irrational) the author employs the "unit height" h, the component of a unit along the axis, and the "unit twist" $t=1/n$, the fraction of a complete rotation for one unit. The parameters h and t vary continuously with n, even if n is irrational. The analysis is readily extended to the case of a coiled-coil, etc. Analysis of the observed diffraction pattern yields the elements of the helical structure. The author states that this more general point of view has led to the elucidation of the helical structures of feather keratin and collagen.

H. A. Hauptman (Washington, D.C.)

11696:

Oda, T.; Naya, S.; Taguchi, I. Matrix theoretical derivation of inequalities. II. Acta. Cryst. **14** (1961), 456–458.

[For Part I by I. Taguchi and S. Naya, see same Acta, **11** (1958), 543–545.] Using a matrix-theoretical representation of the Fourier series which represents the electron density in a crystal, the authors succeed in diagonalizing a modified Karle-Hauptman matrix. Thus they are led to a new derivation of the Karle-Hauptman inequalities. The importance of their analysis is that it may be modified to yield the general set of inequalities involving structure factors for any space group.

H. A. Hauptman (Washington, D.C.)

11697:

Vainshtein, B. K. New equalities coupling structure factors. Soviet Physics. Dokl. **124** (4) (1959), 12–15 (87–90 Dokl. Akad. Nauk SSSR).

New equalities relating the crystal structure factors which, however, are closely related to the Sayre equation, are derived. They suffer from the same limitation as the latter in that preliminary values of a number of phases are required before the values of unknown phases may, in practice, be determined.

H. A. Hauptman (Washington, D.C.)

FLUID MECHANICS, ACOUSTICS

See also 11557, 11664.

11698:

Vinokur, M. Kinematic formulation of rotational gas flow. J. Fluid Mech. **9** (1960), 533–542.

The kinematic formulations for both incompressible and compressible flows are given. It is shown that, in a steady rotational flow, a knowledge of the velocity field also determines the local velocity of sound. The theory is extended to cover the cases in which the local velocity of sound and the specific enthalpy are dependent (as in supersonic flow) and those in which they are independent (as in hypersonic flow). The equations derived can be used to study flows by means of inverse methods.

A. W. Babister (Glasgow)

11699:

Byušgens, S. S. The geometry of unsteady flow of a perfect incompressible fluid. Izv. Akad. Nauk SSSR. Ser. Mat. **24** (1960), 171–202. (Russian)

The author extends the investigations undertaken in his paper [same Izv. **12** (1948), 481–512; MR **10**, 633] to cases of unsteady flow, so that the functions now depend on the time t. As in the previous paper $dM=\omega_0{}^a I_a$, $dI_1=rI_2-qI_3$, etc., with $(\omega_0{}^1)'=r\omega_0{}^2-q\omega_0{}^3$, etc., $p'=rq$, $q'=pr$, $r'=qp$, $\delta=\omega_0{}^1\omega_0{}^2\omega_0{}^3\neq 0$, but now $p=p_a\omega_0{}^a+p_4dt$, $q=q_a\omega_0{}^a+q_4dt$, $r=r_a\omega_0{}^a+r_4dt$. The basic equations of hydrodynamics are now derived for this general system of reference. When $\bar V=V^aI_a$ is the velocity vector, and the vortex vector is $\bar\omega=\frac{1}{2}\operatorname{rot}\bar V=\omega^aI_a$, then $d\bar V=\tilde V^aI_a$, $\tilde V^1=dV^1-V^2r+V^3q$, cycl., $\tilde\omega^1=d\omega^1-\omega^2r+\omega^3q$, cycl. The equations of Helmholtz can now be written

$$\tilde\omega^1\delta=\tilde{\tilde\Omega}\tilde V^1dt-\tilde{\tilde V}\tilde\omega^1dt, \text{ cycl.};$$

here $\delta=\omega_0{}^1\omega_0{}^2\omega_0{}^3\neq 0$, $\tilde{\tilde V}=V^1\omega_0{}^2\omega_0{}^3+V^2\omega_0{}^3\omega_0{}^1+V^3\omega_0{}^1\omega_0{}^2$, $\tilde{\tilde\Omega}=\omega^1\omega_0{}^2\omega_0{}^3+\omega^2\omega_0{}^3\omega_0{}^1+\omega^3\omega_0{}^1\omega_0{}^2$. From these and related formulas a great number of geometrical properties are derived, in which the concept of the vortical plane plays a role. Such a plane, at each point, is the plane which contains the direction of the velocity as well as that of the vortex. Studied are the family of surfaces of constant energy $H=\frac{1}{2}V^2+U+\int dp/\rho$, the relation of streamlines and trajectories, and several other geometrical features. *D. J. Struik* (Cambridge, Mass.)

11700:

Gundersen, Roy. Simple wave flow in ducts. Phys. Fluids **2** (1959), 680–687.

The equations of one-dimensional unsteady flow are applied to a perfect gas in a long tube with uniform cross-section E_0 for $x\leqq 0$ and slowly changing cross-section $E_0+E_1(x)$ for $x\geqq 0$. The actual flow is (approximately) representable as a perturbation of a strictly "plane" basic flow with particle and sound speeds u_0 and c_0. If the basic flow and the perturbation is assumed as isentropic, the two equations for the first-order Riemann "invariants" A, B are of first order in x and t and linear, with coefficients depending on u_0, c_0 and their x-derivatives, the inhomogeneous terms depending on dE_1/dx. $u_0(x,t)$, $c_0(x,t)$ are first chosen as a rarefaction wave centered at $x=t=0$ (produced, e.g., by the sudden withdrawal of a piston), and $E_1(x)=\nu x$, $b\geqq x\geqq 0$. The general solution for A and B includes two arbitrary functions whose determination is simple if the simple wave is complete. For this case A and B have been completely worked out (the γ-value 5/3 requires separate treatment). The result agrees with Chester's two-dimensional treatment [Quart. J. Mech. Appl. Math. **7** (1954), 247–256; MR **16**, 85]. The same problem is solved for $E_1(x)=\nu(x-a)^2$, $0\leqq x\leqq 2a$, and an arbitrary simple wave is also treated for both configurations.

G. Kuerti (Cleveland, Ohio)

11701:

Feindt, E.-G. Berechnung der Strömung des Tandemgitters mit bewegter zweiter Schaufelreihe (2. Hauptaufgabe). Ing.-Arch. **30** (1961), 88–95.

The paper considers the two-dimensional flow of a perfect fluid through two successive cascades in tandem, as encountered in turbomachines. The cascades are in relative (uniform) motion with respect to each other. The contour of the blades and spacing are prescribed. The vorticity in the wake of the profiles, ensuing from the non-stationary character of the flow is neglected. No justification is offered for this assumption.

The relation $\zeta^*=\exp(2\pi\zeta/t)$ maps the physical ζ-plane into a plane in which only two contours appear, provided the spacing t of the two cascades is the same. Through additional conformal mapping, the exterior of the blades is mapped into the interior of a rectangle. The boundary-value problem is solved by means of a suitable distribution of sources and vortices. The results are applied in detail to the case of two cascades consisting of flat plates parallel to the axis. *Rudolf X. Meyer* (Los Angeles, Calif.)

11702:

Papon, André. ★Sur le calcul des grilles de profils. Publ. Sci. Tech. Ministère de l'Air, Bull. Serv. Tech. No. 125, Paris, 1960. xii+62 pp. 15.00 NF.

Ce travail est essentiellement consacré à la détermination et à l'étude de profils de grilles planes en régime incompressible.

L'auteur s'appuie sur une méthode déjà ancienne de Lighthill [Ministry of Supply [London], Aeronaut. Res. Council, Rep. and Memoranda no. 2104 (8766) (1945); MR 8, 610].

Quelques exemples concrets sont traités. Des essais effectués en soufflerie montrent l'intérêt de tels calculs.

P. Germain (Paris)

11703:

Barua, S. N. Some aspects of rotational flow. Proc. Sympos. Mech. Real Fluids (Kharagpur, 1958). J. Sci. Engrg. Res. **3** (1959), 351–356.

11704:

Aggarwal, R. R. Oscillating aerofoil in compressible subsonic flow. Proc. Sympos. Mech. Real Fluids (Kharagpur, 1958). J. Sci. Engrg. Res. **3** (1959), 377–384.

Author's summary: "The paper presents a review of the work on two-dimensional and three-dimensional oscillating aerofoil theory in the case of compressible subsonic flow. Starting with Euler's equations, the fundamental equations in the theory have been established. Various steps leading to the calculation of two-dimensional aerodynamic derivatives have also been given."

11705:

Wilby, Peter G. An experimental and theoretical investigation of second-order supersonic wing-body interference, for a non-lifting body with wings at incidence. Flygtekn. Försöksanstalt. Rep. No. 87 (1960), 16 pp. (German and French summaries)

La théorie de Landahl, Drougge, Beane [J. Aerospace Sci. **27** (1960), 694–702; MR **22** #3345] permettant d'estimer l'effet d'interaction aile fuselage aux nombres de Mach modérément hypersonique ($M\sim 4$) est comparé avec l'expérience pour quelques configurations particulières. L'accord est d'autant meilleur que le nombre de Mach est plus élevé et que la flèche de l'aile est plus faible.

J. P. Guiraud (Meudon)

11706:

Chaudhuri, S. N. A brief survey of the non-linear wing theories. Proc. Sympos. Mech. Real Fluids (Kharagpur, 1958). J. Sci. Engrg. Res. **3** (1959), 363–376.

11707:

Krasovskiĭ, Yu. P. The existence of aperiodic flows with free boundaries. Dokl. Akad. Nauk SSSR **133** (1960), 768–770 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 684–686.

This is a brief summary of results obtained by the author; no proofs are given. The flows are steady, plane and irrotational; and the non-linear condition of constant pressure is satisfied on the free boundary. The profile of the rigid bottom is prescribed, and is horizontal except for a section of finite length. Then the existence of a free-boundary flow can be proved for sufficiently high Froude numbers. When the stream lines are symmetrical with respect to a vertical line, a more precise result can be proved. It can also be shown that solitary waves cannot exist in infinitely deep water, or when the Froude number is less than 1. *F. Ursell* (Cambridge, England)

11708:

Cohen, M. J. Two-dimensional gas jets. J. Appl. Mech. **27** (1960), 603–608.

The author proposes an approximate method of solving an equation of gas dynamics in the hodograph plane. The well-known hodograph coordinates (λ, θ) are associated with the new pair of coordinates $(\Lambda = \ln \lambda^{-1}, \theta)$; the variables introduced are $W = \phi + i\psi$, $\bar{z} = \Lambda - i\theta$; with this the hodograph equation takes a form of some sort of Poisson's equation. The first approximation to the solution is obviously a solution of the Laplace equation; the second and higher approximations can be found by an iteration process, although the result so derived loses more from the increased mathematical complexity than it gains from additional accuracy. The integral which appears in the author's second approximation is integrated by parts in an infinite series which is absolutely convergent everywhere in the z-plane except in small regions near the singularities. The technique is applied to the two-dimensional gas jet. Here the exact solution was given by A. Chaplygin in 1904, and the author shows that his solution approximates closely the Chaplygin solution. Consecutively, the following items are solved: boundary conditions, free streamline boundary in the physical plane and contraction ratio (τ) of two-dimensional jets. The values of τ are tabulated; they show the closeness of the approximation even at the limiting value of $M = 1$. The error (as compared with Chaplygin's solution) is everywhere less than 2 percent. Demtchenko (in 1932), using an approximation of the form of the Kármán-Tsien formula, obtained a result with an error over twice as large as that recorded here.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

11709:

Whitham, G. B. A note on group velocity. J. Fluid Mech. **9** (1960), 347–352.

The kinematic approach to group velocity given in Lighthill and Whitham [Proc. Roy. Soc. London Ser. A **229** (1955), 281–316; MR **17**, 309] is extended to cover the general three-dimensional case. The ideas have particular bearing on the reviewer's theory [J. Fluid Mech. **9** (1960), 333–346; MR **22** #6264] for treating steady wave patterns on non-uniform steady fluid flows.

The author asserts that all the main ideas contained in the present interesting note are implicit in §§ 66 and 67 of the book by Landau and Lifshitz, but the reviewer believes that in making this assertion the author is too modest. *F. Ursell* (Cambridge, England)

11710:

Lehman, R. Sherman. The uniqueness question for waves against an overhanging cliff. Quart. Appl. Math. **18** (1960/61), 271–284.

Water waves against a cliff overhanging at an angle of 135° are treated. The equation is $\Delta\phi - k^2\phi = 0$ for the sector $-3\pi/4 \leq \theta \leq 0$, satisfying $\partial\phi/\partial y - \phi = 0$ for $\theta = 0$, $\partial\phi/\partial n = 0$ for $\theta = -3\pi/4$, where $x = r\cos\theta$, $y = r\sin\theta$, with the reasonable bounds to insure uniqueness

$$(1) \qquad N(\phi) \equiv |\phi| + \left|\frac{\partial\phi}{\partial x}\right| + \left|\frac{\partial\phi}{\partial y}\right| < M \quad \text{for } r > R_0,$$

$$(2) \qquad N(\phi) < Cr^{-7/3+\epsilon} \quad \text{for } r \to 0.$$

By using a reflection principle, some simple interior estimates for derivatives and a Fourier expansion in θ of a related solution the author shows that "For $k < 1$ there are two linearly independent solutions satisfying the conditions (1) and (2). On the other hand for $k \geq 1$, there is only one linearly independent solution satisfying the conditions (1) and (2) and it has a logarithmic singularity at the origin. For $k = 1$ with $y = 0$ this solution approaches a constant for $x \to +\infty$; for $k > 1$ the solution dies out exponentially as $x \to \infty$. In every case the solutions die out exponentially as $y \to -\infty$." *E. Isaacson* (New York)

11711:

Rogožin, V. S. Finding the shape of a body from a given impact pressure. Prikl. Mat. Meh. **23** (1959), 589–591 (Russian); translated as J. Appl. Math. Mech. **23**, 834–838.

The problem considered is that of determining the shape of a body in terms of the impact pressure when the body is in contact with the surface of a perfect fluid. The body is permitted to have both a linear motion and an angular velocity. It is assumed that the body has infinite length and uniform cross-section and that the fluid has infinite depth. Thus the flow is two-dimensional and the velocity potential and stream function determine an analytic function. Three special cases are treated and in each case the problem is solved by choosing an appropriate conformal mapping. *A. H. Copeland, Sr.* (Ann Arbor, Mich.)

11712:

Yen, Kuo-Tai. Optimal coordinates for a simple shear flow over a flat plate. Z. Angew. Math. Phys. **11** (1960), 228–236. (German summary)

The author repeats the problem first treated by T. Y. Li [J. Aero. Sci. **22** (1955), 651–652, 724–725; MR **17**, 99, 205] in parabolic coordinates. *Y. H. Kuo* (Peking)

11713:

Bourrières, François-Joseph. ★Équations intrinsèques du mouvement à trois dimensions des fluides à viscosité. Publ. Sci. Tech. Ministère de l'Air, No. 368, Paris, 1960. viii + 67 pp. 16.50 NF.

The author expresses the equations of motion and continuity for the steady three-dimensional flow of an

incompressible, viscous fluid in intrinsic forms. That is, these equations are expressed in terms of the curvatures and torsions of three orthogonal congruences of curves ((1) the streamlines; (2) curves determined by the principal normals and binormals of the streamlines) and the directional derivatives of pressure and magnitude of the velocity along these directions. Particular attention is given to the case when the streamlines possess ∞^1 surfaces orthogonal to these curves. Here, the normal and geodesic curvatures of the above congruences are introduced. The computations are explicitly determined for each component of the equations of motion.

N. Coburn (Ann Arbor, Mich.)

11714:

Kotlyar, Ya. M. An approximation for the Reynolds equation. Dokl. Akad. Nauk SSSR **130** (1960), 41–44 (Russian); translated as Soviet Physics. Dokl. **5**, 31–35.

A method is given for approximating, by recurrent functions, the solution of the "Reynolds" equation

$$\frac{\partial}{\partial x}\left(Q^2\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(Q^2\frac{\partial P}{\partial y}\right)=0,$$

where $Q=Q(x)$. Two applications are made to the hydrodynamic theory of lubrication.

L. M. Milne-Thomson (Madison, Wis.)

11715:

Pell, W. H.; Payne, L. E. The Stokes flow about a spindle. Quart. Appl. Math. **18** (1960/61), 257–262.

The Navier-Stokes equations are solved for slow steady flow past a spindle. Since the terms representing acceleration are neglected, the equations dealt with are linear, and can be reduced (by use of the continuity equation) to the form

$$\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial x^2}\right)^2\psi=0,$$

in which ψ is Stokes' stream function, and r and x are two of the cylindrical coordinates. The solution is given in integral form, in which conal functions appear. The key steps in the solution are: (1) the use of Payne's theorem (which is very similar to Almansi's theorem for solutions of the differential equation $\nabla^{2n}\psi=0$, $n=$integer); and (2) the use of bipolar coordinates.

The solution is a welcome addition to the family of known Stokes flows.

Chia-Shun Yih (Ann Arbor, Mich.)

11716:

Yudovič, V. I. Periodic motions of a viscous incompressible fluid. Dokl. Akad. Nauk SSSR **130** (1960), 1214–1217 (Russian); translated as Soviet Math. Dokl. **1**, 168–172.

Viene dato un teorema di esistenza di soluzioni periodiche per il sistema di Navier-Stokes. Precisamente, si considera il sistema: $\partial\mathbf{v}/\partial t+(\Sigma_i v_i\partial/\partial x_i)\mathbf{v}-\Delta\mathbf{v}=\mathbf{f}-\operatorname{grad}p$, $\operatorname{div}\mathbf{v}=0$, per $x\in\Omega$ (insieme aperto e limitato di frontiera S) con la condizione al contorno $\mathbf{v}|_s=\boldsymbol{\alpha}$. Si suppone che $\mathbf{f}$ ed $\boldsymbol{\alpha}$ siano periodiche rispetto a t di periodo T e si cercano soluzioni $\{\mathbf{v},p\}$ periodiche con il medesimo periodo T. Viene dimostrata l'esistenza di una soluzione sotto le ipotesi: che S ed $\boldsymbol{\alpha}$ siano sufficientemente regolari, che si abbia, per ogni componente S_k della frontiera, $\int_{S_k}\boldsymbol{\alpha}\cdot\mathbf{n}ds=0$ ($\mathbf{n}$ versore normale ad S) e che sia $\sup_t\|\mathbf{f}\|_{L^2(\Omega)}<\infty$.

Il metodo (che viene, in termini generali, enunciato per una classe di equazioni differenziali non lineari in uno spazio di Hilbert) è una variante del metodo di Faedo-Galerkin-Hopf: vengono considerate le soluzioni periodiche del sistema diffrenziale approssimante, quindi si passa al limite. L'esistenza di una soluzione periodica nel caso 2-dimensionale è stata data contemporaneamente anche dal recensore [Prodi, Rendi, Sem. Mat. Univ. Padova **30** (1960), 1–15; MR **22** #5823].

L'A. dà poi teoremi di regolarizzazione per la soluzione: nel caso 2-dimensionale se $\mathbf{f}\in L^r(Q_T)$ (essendo Q_T il cilindro $\Omega\times[0,T]$) si ha $\partial\mathbf{v}/\partial t$, $\Delta\mathbf{v}\in L^r(Q_T)$ (con analoghi risultati per le derivate di ordine superiore). Nel caso 3 dimensionale risulta: $\partial\mathbf{v}/\partial t$, $\Delta\mathbf{v}\in L^{5/4}(Q_T)$. Di questi risultati di regolarizzazione (che sono senza dubbio i più interessanti) non viene data alcuna dimostrazione.

G. Prodi (Trieste)

11717:

Segel, Lee A. Application of conformal mapping to viscous flow between moving circular cylinders. Quart. Appl. Math. **18** (1960/61), 335–353.

Some two-dimensional viscous motions between two eccentric circular cylinders, of which the outer is fixed, are considered. The main problems are the slow rotation and the vibration of the inner cylinder; the eccentricity is supposed small, and the area of the cross-section is represented conformally on the area between two concentric circles.

W. R. Dean (London)

11718:

Scholz, N. Ergänzungen zum Grenzschichtquadraturverfahren von E. Truckenbrodt. Ing.-Arch. **29** (1960), 82–92.

Das von E. Truckenbrodt [Ing. Arch. **20** (1952), 211–228] entwickelte Quadraturverfahren zur Bestimmung der laminaren und der turbulenten Grenzschicht für die ebene und rotationssymmetrische Strömung bei glatter Wand und beliebigem Druckanstieg wird erweitert und in der praktischen Handhabung weiter vereinfacht. Die Erweiterung erstreckt sich auf die Behandlung rauher Oberflächen und auf die Berücksichtigung der mit der Aufheizung der Grenzschicht verbundenen Änderung der Stoffeigenschaften des strömenden Mediums. Die Änderung der Stoffeigenschaften wird näherungsweise mit einer mittleren Grenzschichttemperatur berücksichtigt, die aus der Reibungswärme und dem Wärmeübergang an der Wand ermittelt wird.

L. Speidel (Mülheim)

11719:

Becker, Ernst. Eine einfache Verallgemeinerung der Rayleigh-Grenzschicht. Z. Angew. Math. Phys. **11** (1960), 146–152. (English summary)

The author studies the compressible boundary-layer flow near an infinite plate which is set into motion at time $t=0$ and moves with the velocity $u_w\sim t^n$. The wall may be porous, and suction or blowing of the type $v_w=v_0(t_0/t)^{1/2}$ may occur. The temperature of the wall is kept constant. The product $(\rho\mu)$ of density and viscosity coefficients is assumed to be constant and the heat due to friction is neglected in the energy equation. With these assumptions similar profiles for velocity and for temperature can be

obtained. The analytic expressions for these profiles are simple. At the end of the paper the influence of frictional heat is considered. *I. Flügge-Lotz* (Stanford, Calif.)

11720:

Jain, A. C. On compressible boundary layer on a flat plate with uniform suction. Proc. Indian Acad. Sci. Sect. A **53** (1961), 12–27.

11721:

Shen, S. F.; Solomon, J. M. First-order slip effects on the compressible laminar boundary layer over a slender body of revolution in axial flow. J. Aerospace Sci. **28** (1961), 508–510.

11722:

Evans, N. A. Heat transfer in the unsteady compressible laminar boundary layer on a flat plate. Proc. 1960 Heat Transfer Fluid Mech. Inst., pp. 77–91. Stanford Univ. Press, Stanford, Calif., 1960.

The author considers a semi-infinite flat plate which moves with velocity $u_w(t)$ parallel to itself into a compressible fluid, starting from rest. After reduction to incompressible form under assumption of a linear viscosity-temperature law, solutions in series-form are obtained for the determination of heat-transfer with prescribed surface enthalpy and surface enthalpy with prescribed heat transfer. The series proceed in powers of $\xi_0 = (x/\int_0^t(-u_w(\tau))d\tau)^{1/2}$, where x is the distance from the leading edge. As usual in investigations of this and similar problems, the series are assumed to converge as long as ξ_0 is small. Universal coefficient functions up to the third order are numerically obtained.

W. C. Rheinboldt (Syracuse, N.Y.)

11723:

Kawamata, Shin. A simplified method calculating the laminar boundary layer. J. Phys. Soc. Japan **16** (1961), 805–807.

Goertler variables are used in an approximate analysis of the steady incompressible boundary layer. The central idea is to consider the "Falkner-Skan parameter" varying slowly in the streamwise direction. Application of the method to several special profiles gives separation points in close agreement with those found by other authors.

L. A. Segel (Troy, N.Y.)

11724:

Yih, Chia-Shun. Instability of a rotating liquid film with a free surface. With an appendix by J. F. C. Kingman. Proc. Roy. Soc. London Ser. A **258** (1960), 63–89. (1 plate)

A film of viscous liquid with surface tension is on the outside of a rotating cylinder. Gravity is neglected. Wave crests on the surface are assumed to lie in circles normal to the axis. Waves travelling along the cylinder are shown to be damped. Stationary waves are always unstable if the wavelength is sufficiently large. Graphs show how, for large Reynolds number, the critical wavelength corresponding to maximum growth rate increases with increasing surface tension or decreasing rotation, and also give the growth rate for given wave number, surface tension, rotation, viscosity, cylinder radius, and film thickness expressed in terms of two non-dimensional numbers for three values of the third. Similar graphs are given for small R, and the surface tension is found to be more important than viscosity for large rotations in determining the instability. Detailed comparison with experiments is made and the effect of gravity and the possibility that the velocity of the fluid picked up from a trough may not be that of the cylinder are neatly discussed.

The appendix shows that if the rotation is increased from zero the most likely disturbance has axial symmetry, but for large steady rotations this is not so.

R. S. Scorer (London)

11725:

Wooding, R. A. Rayleigh instability of a thermal boundary layer in flow through a porous medium. J. Fluid Mech. **9** (1960), 183–192.

A heated liquid rises slowly through a semi-infinite porous medium towards a horizontal surface where it mixes with a layer of cool overlying fluid. Assuming a steady uniform motion in the medium and constant thermal diffusivity, there is a thermal boundary layer of an exponential form. The stability of this steady-state distribution to small disturbances is considered in this paper. Under appropriate physical assumptions and the Boussinesq approximation it is shown that the thermal boundary layer is stable provided that the Rayleigh number does not exceed a critical positive value corresponding to a non-zero value of the wave number. Both a series method and the method of Chandrasekhar [Mathematika **1** (1954), 5–13; MR **16**, 84] are used to solve the eigenvalue problem. The paper concludes with an interesting qualitative discussion of the problem based on physical considerations.

R. C. DiPrima (Troy, N.Y.)

11726:

Reynolds, Alan. On the dynamics of turbulent vortical flow. Z. Angew. Math. Phys. **12** (1961), 149–158. (German summary)

From author's summary: "A turbulent compressible vortex flow is examined. The equations of motion are simplified by neglecting small terms. Experimental data permit a determination of the error induced in a particular case. An important point in the simplification is the assumption that radial gradients are much larger than axial gradients. For strong vortex flows both experiment and existing theoretical considerations indicate that this assumption is well founded. The simplified equations display several important aspects of flows with high swirl, and provide a basis for further work."

J. L. Lumley (University Park, Pa.)

11727:

Dem'yanov, Yu. A.; Šmanenkov, V. N. Investigation of reverse flows in the region of separation of the turbulent boundary layer. Prikl. Mat. Meh. **24** (1960), 237–239 (Russian);; translated as J. Appl. Math. Mech. **24**, 340–343.

A separated region occurs in the flow up a step or on a sting attached to the nose of a blunt body.

The flow in such a region is assumed to be a turbulent mixing zone attached to a semi-infinite free stream. A simple solution is found by using free turbulence theory,

and agreement with experimental results is found for the angle of the separated flow in the two cases mentioned above. *L. F. Crabtree* (Farnborough)

11728:

Pantchev, Stoïtcho. Nouveau système d'équations dynamiques de la turbulence isotrope. C. R. Acad. Sci. Paris **251** (1960), 1859–1861.

Se plaçant dans un écoulement turbulent isotrope, utilisant l'hypothèse habituelle de quasi-normalité des corrélations d'espace, et négligeant certaines corrélations triples, l'auteur établit une équation aux dérivées partielles vérifiée par le scalaire G qui définit les corrélations d'espace, et une seconde équation aux dérivées partielles entre G et la fonction de corrélation de la température. *J. Bass* (Paris)

11729:

Depassel, Roger. ★Écoulement de l'air à grande vitesse dans un tuyau de section circulaire. Publ. Sci. Tech. Ministère de l'Air, No. 360, Paris, 1960. x+98 pp. 14.50 NF.

From author's summary: "On étudie l'écoulement de l'air à vitesse subsonique dans des conditions sensiblement adiabatiques. Etude théorique de la couche limite laminaire (extension de la méthode de Gruschwitz), de la couche limite turbulente avec profil de vitesse en loi de puissance et exposant variable au long de l'écoulement en utilisant les trois équations intégrales fondamentales, et de l'écoulement établi. La méthode approchée de Young et Winterbottom est améliorée. Ces théories sont comparées aux valeurs expérimentales obtenues dans un montage particulier." *D. A. Spence* (Farnborough)

11730:

Buleev, N. I. Distribution of velocity and temperature in turbulent flow of a liquid in a round pipe. Voprosy teploobmena, pp. 208–232. (Russian) Izdat. Akad. Nauk SSSR, Moscow, 1959.

11731:

Nevzglyadov, V. G. Thermodynamics of turbulent systems. Ž. Éksper. Teoret. Fiz. **39** (1960), 1727–1733 (Russian. English summary); translated as Soviet Physics JETP **12** (1961), 1206–1210.

An attempt to generalize the second law for turbulent systems by introducing a turbulent entropy S_e depending both on the average internal energy and the turbulent energy. Explicit expressions for flux density and rate of production of S_e are given. *L. J. F. Broer* (Delft)

11732:

Oguchi, Hakuro. Blunt body viscous layer with and without a magnetic field. Phys. Fluids **3** (1960), 567–580.

Cet article apporte une contribution au problème du transfert de chaleur au voisinage du point d'arrêt d'un véhicule hypersonique en atmosphère modérément raréfiée. La solution est du type "viscous layer" suivant la terminologie de Probstein [*Rarefied gas dynamics: Proc. 1st Internat. Sympos.*, Pergamon, New York, 1960; p. 416]. Autrement dit, toute la couche de choc est supposée relever des équations de Navier-Stokes, les conditions de Rankine-Hugoniot étant toutefois retenues sur le choc, qui est supposé sphérique ainsi que l'obstacle. Une hypothèse de similitude conduit à des équations différentielles qui sont les mêmes que chez Probstein, la différence tenant à ce qu'ici la solution est explicite au lieu d'être purement numérique. Il s'agit d'une solution approchée (par rapport à Probstein) obtenue par développement en $\varepsilon=(\gamma-1)/(\gamma+1)$. Le résultat remarquable et quelque peu inattendu, est un excellent accord avec la solution "exacte", même si l'on ne retient que les termes d'ordre ε, pourvu que l'on soit en atmosphère modérément raréfiée ($R_e \sim 10^3$). Comme il se doit, cette circonstance favorable se déteriore quand R_e devient très grand. L'auteur montre comment cela est du à un comportement singulier du double passage à la limite $\varepsilon \to 0$, $R_e \to \infty$. Accessoirement, l'influence d'un champ magnétique, créé par un dipole, est discutée. *J. P. Guiraud* (Meudon)

11733:

Imšennik, V. S. The isothermal scattering of a gas cloud. Dokl. Akad. Nauk SSSR **131** (1960), 1287–1290 (Russian); translated as Soviet Physics. Dokl. **5**, 253–256.

A solution of the one-dimensional equation of gas dynamics is obtained, but the equations being solved are not given. The gas is being irradiated and is absorbing energy. The temperature of the cloud remains constant and the entropy of the gas increases. A layer of gas is then considered in which there are two regions, one where there is an incident isothermal expansion wave and another where there are waves reflected from the center of the layer. *G. C. McVittie* (Urbana, Ill.)

11734:

Napolitano, L. G.; de Socio, L. Moti instazionari omoentropici de miscele reagenti in equilibrio chimico. Aerotecnica **40** (1960), 172–177.

Un procédé de calcul est indiqué pour la fonction de Riemann dans le cas d'un gaz en équilibre chimique. *J. P. Guiraud* (Paris)

11735:

Tomilov, E. D. On a certain form of exact particular solutions of a plane, vortex free gas flow. Prikl. Mat. Meh. **24** (1960), 537–540 (Russian); translated as J. Appl. Math. Mech. **24**, 783–789.

A family of hodograph solutions is found in which ψ is a polynomial of arbitrary degree n in θ (the direction angle of the velocity) with coefficients depending on the speed. The case $n=1$ gives a subsonic stream bounded in part by a semi-infinite straight wall and in part by a curved streamline (springing from the end of the wall) on which the speed is sonic. *T. M. Cherry* (Melbourne)

11736:

Helliwell, J. B. Unsymmetrical flow patterns past a finite wedge profile in a high subsonic stream. Proc. Cambridge Philos. Soc. **57** (1961), 401–414.

The flow pattern past a thin wedge-like profile set at a small angle of attack in a gas flowing with high subsonic or sonic velocity is discussed, on the transonic approximation, by the hodograph method. In the model considered the flow

has a stagnation point at the nose of the wedge and breaks away, with velocity equal to that of sound, from the shoulders, whose distances from the nose are slightly unequal. The solution of the boundary-value problem for the wedge in a channel is formulated as a pair of dual integral equations, and the complete solution is given for the wedge in a free stream. The lift coefficient is calculated in terms of the transonic similarity parameter.

T. M. Cherry (Melbourne)

11737:

Murgulescu, Elena. De la théorie des mouvements coniques dans le cas des ailes minces, de section donnée. Acad. R. P. Romîne. Stud. Cerc. Mat. **11** (1960), 61–94. (Romanian. Russian and French summaries)

In this paper, the supersonic flow around a thin angular wing is studied within the theory of conical flows. The problem to be solved is the determining of a function of a complex variable, the real part of which represents the component of the disturbance velocity normal to the plane in which the wing is assumed to be contained. All possible cases are considered, i.e., the wing with both leading edges subsonic, with one subsonic leading edge and the other supersonic and with both leading edges supersonic. The method of approach is a unified one and consists of reducing the problem to that of Dirichlet for a plane having straight-line segments situated on the real axis, as a boundary. If the wing has subsonic leading edges, the expressions of the solutions appear as sums of functions, some of which are solutions of some particular problems, and the others are conveniently chosen to satisfy the boundary conditions. Then the case of the wings with subsonic trailing edges is treated, the existence of a vortex sheet detaching from the trailing edge being assumed.

T. Oroveanu (Bucharest)

11738:

Carrière, Pierre. Effet d'une injection de fluide dans l'eau morte, sur les conditions de recollement d'un écoulement plan supersonique. C. R. Acad. Sci. Paris **251** (1960), 2877–2879.

Author's summary: "Un écoulement décollé de Mach. M_1 subit au moment du recollement une déviation Ψ qui dépend d'un paramètre K lié à la distribution de vitesse dans la couche de mélange. La variation δK de ce paramètre sous l'effet d'une injection de fluide dans l'eau morte et la variation $\delta\Psi$ qui en résulte s'expriment en fonction du débit et de la quantité de mouvement injectés. La forme très générale de l'expression obtenue est confirmée par l'expérience."

11739:

Sedney, Raymond. Some aspects of nonequilibrium flows. J. Aerospace Sci. **28** (1961), 189–196, 208.

"Non-equilibrium" here refers to the departure from the local molecular equilibrium distribution due to the vibrational excitation lag of the molecules. The relatively slow adjustment of the vibrational degrees of freedom is an irreversible thermodynamic process, hence with entropy rise. After a brief discussion of some thermodynamic relations of a vibrational relaxing diatomic gas, an equation relating the entropy gradient to the vorticity for nonequilibrium flows is derived which can be considered as a generalization of Crocco's theorem for the corresponding equilibrium flows. The characteristic equations are derived as usual for supersonic flows from the equations of motion which are of the hyperbolic type. These equations are used in treating two particular flows; namely, two-dimensional flow over (i) a cusped body which has a straight shock wave, and (ii) a wedge to which the attached shock becomes curved on account of the vibrational relaxation time.

Vi-Cheng Liu (Ann Arbor, Mich.)

11740:

Marsh, H. W. Exact solution of wave scattering by irregular surfaces. J. Acoust. Soc. Amer. **33** (1961), 330–333.

Scattering from a corrugated surface is studied by Wiener's generalized harmonic analysis.

A. E. Heins (Ann Arbor, Mich.)

11741:

Satô, Yasuo. Normal mode interpretation of the sound propagation in whispering galleries. Nature **189** (1961), 475–476.

11742:

Brehovskih, L. M.; Eliseevnin, V. A. Wave propagation in an inhomogeneous waveguide. Akust. Ž. **6** (1960), 284–291 (Russian); translated as Soviet Physics. Acoust. **6** (1961), 282–290.

Propagation of waves is considered in a medium with wave number given by

$$k^2(x, z) = k_0^2[(1-a)/\cosh^2(z/H)+b\tanh(x/L)+a].$$

When $0<a<1$, such a medium may be considered to be a waveguide of height H, with axis in the plane $z=0$, which is homogeneous for $|x|\gg L$. The region $|x|\lesssim L$ is a transition region in which the medium parameters vary significantly in the horizontal as well as in the vertical direction. The wave equation with this wave number is separable, the solutions being products of spherical and exponential functions. For the normal modes, the reflection coefficient, phase velocity and group velocity are calculated. The problem is also considered from the standpoint of ray theory, conditions being determined under which sound rays have a given type of behavior.

R. N. Goss (San Diego, Calif.)

11743:

Wilcox, Calvin H. A variational method for computing the echo area of a lamina. Boundary problems in differential equations, pp. 301–314. Univ. of Wisconsin Press, Madison, Wis., 1960.

This paper concerns the diffraction of a time-harmonic acoustic plane wave by a rigid curved lamina. The corresponding boundary-value problem is solved in terms of the solution of a functional equation, due to A. W. Maue [Z. Physik **126** (1949) 601–618; MR **11**, 293]. Starting from this equation, the author sets up a variational expression for the echo area of the lamina and indicates how to obtain approximate values of this area. Application to a specific example is not included.

C. J. Bouwkamp (Eindhoven)

11744:

Nomura, Yûkichi; Osanai, Tadao. Diffraction of plane sound wave by many equal circular holes arbitrarily distributed in an infinitely large rigid plane plate. J. Phys. Soc. Japan **16** (1961), 819–831.

Following the work of Nomura in solving the problem of plane wave diffraction by a circular disk or circular hole by a method involving cylindrical polar coordinates and series of hypergeometric functions, the authors have treated the case of two equal circular holes in a plane screen. In this paper they extend the method to consider an arbitrary distribution of equal non-overlapping holes, arbitrarily distributed in a plane screen. The formulae involving doubly infinite series are derived for various physical quantities, and for the normal incidence case, approximate results for low frequencies are given.

V. M. Papadopoulos (Melbourne)

11745:

Kolmer, F.; Tichý, J. Energy relations in sound propagation in porous materials with rigid skeleton. Czechoslovak J. Phys. **10** (1960), 872–879. (Russian summary)

Authors' summary: "The paper deals with the determination of the frequency dependence of the acoustic resistance, of the structure factor, the porosity factor and the constant giving whether the process in the propagation of sound in porous materials with a rigid skeleton is isothermal, adiabatic or polytropic. The latter dependence enables a conclusion to be reached on the energy relations during sound propagation in porous materials.

"The derivation of the wave resistance and of the constant of wave propagation in a porous material with a rigid skeleton is given and a method described for calculating the constants characterizing the material on the basis of measurements of the wave resistance and the acoustic impedance of the material. The calculation was carried out for felt and it was found that the acoustic resistance and structure factor depends on the frequency and that the process at low frequencies approaches the isothermal and at high frequency the adiabatic. It is shown that the structure factor is not equal to unity even in order of magnitude, as is often assumed in the literature. It is shown that for a complete knowledge of the acoustic resistance necessary for the calculations it is not enough to determine it by the static method."

11746:

Krasil'nikov, V. N. Effect of a thin elastic layer on the propagation of sound in a liquid half space. Akust. Ž. **6** (1960), 220–228 (Russian); translated as Soviet Physics. Acoust. **6**, 216–224.

Author's summary: "In the present paper the effect of a thin elastic layer on acoustical processes in a liquid half-space at low frequencies is discussed. It is shown that a point source excites a surface flexural wave at the boundary between the half-space and plate. The properties of this wave are analyzed, and it is shown that under definite conditions it is the principal part of the sound field near the surface."

E. J. Scott (Urbana, Ill.)

11747:

Marsh, H. Wysor; Schulkin, M.; Kneale, S. G. Scattering of underwater sound by the sea surface. J. Acoust. Soc. Amer. **33** (1961), 334–340.

Theoretical and observational results in the quantitative description of ocean waves, the scattering of sound by rough surfaces and sound propagation models are combined to obtain results on the scattering of sound by the sea surface. Some of the statements concerning ocean waves will probably be corrected with further research. Also the assumption of an isotropic sea surface can probably be removed to obtain some directional effects.

W. J. Pierson, Jr. (New York)

11748:

Buldyrev, V. S. The investigation of exact solutions of nonstationary diffraction problems in the neighborhood of wave front. Dokl. Akad. Nauk SSSR **129** (1959), 291–294 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1218–1221.

In this paper the author investigates the convergence of an expansion obtained in previous papers in connection with the solutions of non-stationary diffraction problems. For isotropic acoustic media separated by a circular cylindrical or spherical boundary, the author shows how to form a power series in the parameter γ which converges near the wave front. In some cases this series represents the field exactly, and in others, with an accuracy up to the term which is an analytic function of the space coordinates and time.

E. J. Scott (Urbana, Ill.)

11749:

Werner, Peter. Zur mathematischen Theorie akustischer Wellenfelder. Arch. Rational Mech. Anal. **6**, 231–260 (1960).

This paper deals in detail with the fundamental problems related to the stationary acoustic field in an inhomogeneous medium. After the separation of the time factor $\exp(-i\omega t)$, one is led to the reduced scalar wave equation (*) $\rho_0\nabla(\rho_0^{-1}\nabla U)+\kappa^2 U=f$, where $\rho_0\,(>0)$, $\kappa^2\,(0\leqq \arg\kappa<\pi/4)$ and f are given functions of the (3-dimensional) space coordinates x. It is assumed that ρ_0 and κ are constant and $f=0$ for sufficiently large $r=|x|$, and U is required to satisfy the Sommerfeld radiation condition (**) $\partial U/\partial r-i\kappa U=o(r^{-1})$, $U=O(r^{-1})$ for $r\to\infty$. Three problems are posed: (*) and (**) are to be solved: (A) in the whole space R^3; (B) in a domain exterior to a bounded closed surface where the Neumann boundary condition is imposed on U; (C) in the presence of a surface of discontinuity for ρ_0 and κ, where certain jump conditions are imposed on U.

This paper contains (1) a detailed physical and mathematical consideration leading to the setup of the fundamental differential equation in space-time variables, (2) the reduction of the problem to the above form by assuming that the motion is stationary and separating the time factor $\exp(-i\omega t)$, (3) introduction of the fundamental solutions for constant ρ_0 and κ satisfying (**), (4) the proof of the uniqueness theorem for the problems (A) to (C) (in which the theorem of Rellich and the unique continuation theorem are used) and (5) the proof of the existence theorem for (A) (in which the problem is converted into an integral equation on a bounded domain and the Riesz-Schauder theorem is applied to the latter). The remaining problems are to be dealt with in subsequent publications.

T. Kato (Tokyo)

11750:

Leslie, C. B.; Sorensen, N. R. Integral solution of the shallow water sound field. J. Acoust. Soc. Amer. **33** (1961), 323–329.

Formulas for the velocity potential for the propagation of low-frequency sound from a point source in a two-layered system of liquids have been developed by several authors [see, e.g., Ewing, Jardetsky and Press, *Elastic waves in layered media*, McGraw Hill, New York, 1957; MR **22** #1475]. The present paper is devoted to giving a computation scheme for the case in which the sound velocity (c_2) in the bottom layer is greater than that (c_1) in the upper layer. This case is the important one for coastal waters; according to the authors, it has not been successfully investigated previously for intermediate ranges. A method of numerical integration is presented which covers the cases of horizontal distances between zero and several times the depth of the upper layer; here the lower layer (e.g., mud) is assumed to have infinite depth. For the case $c_2 > c_1$ considered here, the integrand of the relevant integral,

$$\varphi_2 = 2\int_0^\infty J_0(kr)\,\frac{\sinh \nu_1 h}{\nu_1} \times \frac{\nu_1 \cosh \nu_1(H-z) + \eta\nu_2 \sinh \nu_1(H-z)}{\nu_1 \cosh \nu_1 H + \eta\nu_2 \sinh \nu_1 H}\,k\,dk$$

where $\eta = \rho_1/\rho_2$, $k_j = \omega/c_j$, $\nu_j = (k^2 - k_j^2)^{1/2}$ $(j = 1, 2)$, $k \le z \le H$, and ω is the angular frequency (the factor $\exp(i\omega t)$ being omitted from the expression φ_2 for the velocity potential in the bottom layer), has simple poles on the real axis, except for very low frequencies. By transformation of the "integral cuts", the problem is reduced to the evaluation of branch line integrals and of the residues at a finite number of poles. These calculations were then made on a high-speed digital computer. A check on the results was obtained by using both this method and that of direct numerical integration for the special case of very low frequencies. A satisfactory agreement was found between the two methods for these frequencies.

D. H. Hyers (Berkeley, Calif.)

11751:

Keck, Winfield; Beyer, Robert T. Frequency spectrum of finite amplitude ultrasonic waves in liquids. Phys. Fluids **3** (1960), 346–352.

The classical problem of the propagation of finite-amplitude sound waves through a fluid medium, which had been considered by Stokes, Earnshaw and Riemann, is re-evaluated in this paper. Using the Lagrange description of the equations of motion, the theory of harmonic plane sound waves of finite amplitude in an inviscid fluid is first discussed. In this inviscid case the authors derive independently the exact solution of E. Fubini-Ghiron [Alta Frequenze **4** (1935), 530]. This exact inviscid solution is then employed in a perturbation analysis, by successive approximations, of the viscous motion. The analysis has been carried out to higher-order terms, so that the location of the maximum of the second harmonic can be evaluated with some accuracy. The results agree well with the experimental data.

T. Yao-tsu Wu (Pasadena, Calif.)

11752:

Tomita, Yukio. Analytical treatments of non-Newtonian fluid flow by introducing the conception of boundary layer. Bull. JSME **4** (1961), 77–86.

The author discusses boundary-layer equations and an approximate solution for a flow in a circular semi-infinite pipe for the so-called power-law fluids.

J. L. Ericksen (Baltimore, Md.)

11753:

Gutkin, A. M. Flow of a viscoplastic medium between rotating disks. Dokl. Akad. Nauk SSSR **134** (1960), 1048–1050 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1105–1107.

The medium has the property that, for shear stress τ greater than a critical value τ_0, the shear strain rate is proportional to $\tau - \tau_0$; for $\tau < \tau_0$, the shear stress appears to be related to the flow solely through the equations of motion. The discs are rotating at the same speed and there is a small radial outflow. The distance between the discs is sufficiently small compared to their radius to enable boundary-layer approximations to be used. {This reviewer is not satisfied that the author has consistently applied his assumptions in the region of flow where $\tau < \tau_0$.}

D. R. Bland (Manchester)

11754:

Narasimhan, M. N. L. Laminar non-Newtonian flow in an annulus with porous walls. Z. Angew. Math. Mech. **41** (1961), 44–54. (German and Russian summaries)

The author considers the steady flow of a non-Newtonian fluid along a tube bounded by concentric circular cylinders. These walls are porous, and a radial component of velocity is introduced by assuming that liquid is injected at one wall and ejected at an equal rate at the other. This gives rise to a cylindrically symmetrical boundary-value problem. Components of velocity, and the pressure are calculated. The corresponding problem is also examined for flow between parallel porous plates.

J. E. Adkins (Providence, R.I.)

11755:

Bhatnagar, P. L. On two-dimensional boundary layer in non-Newtonian fluids with constant coefficients of viscosity and cross-viscosity. Proc. Indian Acad. Sci. Sect. A **53** (1961), 95–97.

The author discusses boundary-layer approximations for a particular type of Reiner-Rivlin fluid.

J. L. Ericksen (Baltimore, Md.)

11756:

Stahanov, I. P.; Stupočenko, E. V. The structure of Mach lines in relaxing media. Dokl. Akad. Nauk SSSR **134** (1960), 1044–1047 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 964–968.

The authors study the linearized equations of relaxation hydrodynamics for plane steady flow past a thin wedge. The case when the Mach number at infinity is greater than one {$M_\infty > 1$; there is a misprint here in the line above (39)} is treated. Assuming boundary conditions, the authors solve the partial differential equation, to which the system reduces, by contour integration. It is shown that in the first approximation a discontinuity is produced close to the vertex and that the intensity of this discontinuity decreases exponentially with the distance from the vertex, as in equilibrium hydrodynamics. However, by use of the second approximation, it is shown that no discontinuity

occurs. Finally, the general problem of linearized disturbances is studied briefly.

N. Coburn (Ann Arbor, Mich.)

11757:

Kahn, F. D. A dispersion relation for waves of finite amplitude in a two-stream plasma. J. Fluid Mech. **10** (1961), 357–365.

A dispersion relation is obtained for plane-charge waves of finite amplitude in a plasma containing two oppositely charged streams of particles. The relation between the wave-length and the maximum variation of the electrostatic potential can be found in a relatively simple parametric form. Two direct expressions for the wave-length in terms of the variation of the potential can also be found, each expression being approximately valid for certain ranges of the amplitude. *G. C. McVittie* (Urbana, Ill.)

11758:

McCune, James E. On the motion of thin airfoils in fluids of finite electrical conductivity. J. Fluid Mech. **7** (1960), 449–468.

The author gives an extension of the investigation by Sears and Resler [same J. **5** (1959), 257–273; MR **21** #551], to the case of finite electrical conductivity (σ). Following them, he considers a plane, slightly disturbed flow $(U+u, v, 0)$ under the action of a slightly disturbed magnetic field $(h_x, H_\infty+h_y, 0)$. The fluid is incompressible and inviscid. The linearized non-dimensional Euler and Maxwell equations now depend on the magnetic Reynolds number $R_m=\sigma UL$ and the ratio $m=(U/\text{Alphén speed})=U(4\pi\rho)^{1/2}/H_\infty$ for $\mu=1$ and can be put in the form (1) $\mathscr{D}\xi=(4\pi R_m)^{-1}\nabla^2\xi_x$, (2) $\mathscr{D}\Omega=(4\pi m^2R_m)^{-1}\nabla^2\xi_y$ with $\mathscr{D}=\partial^2/\partial x^2-m^{-2}\partial^2/\partial y^2$. Here ξ and Ω are the (only non-vanishing) z-components of curl **h** and curl **q**. Equation (1), which appeared earlier [loc. cit.], is within $O(R_m^{-1})$ identical with the equation for plane damped sound waves.

The key to the present study is the solution of (1): (3) $\xi_\lambda=A_\lambda e^{i\lambda x}e^{\mp\lambda my}$, where

$$\tau^2=(i\lambda/4\pi R_m-1)/(i\lambda m^2/4\pi R_m+1)$$

and the wave number λ is arbitrary. The associated solutions for the disturbance fields in $y>0$ are $\{u_\lambda, v_\lambda; h_{x\lambda}, h_{y\lambda}\}=\{\alpha B_\lambda, \beta B_\lambda; \alpha A_\lambda, \beta A_\lambda\}e^{i\lambda x}e^{-\lambda my}+\{\phi_{1,x}, \phi_{1,y}; \phi_{2,x}, \phi_{2,y}\}$; here ϕ_1 and ϕ_2 are harmonic; α and β are constants depending on λ, m, R_m; $B_\lambda=(i\tau/m)A_\lambda$; A_λ is arbitrary; and the real part is to be taken. The irrotational fields given by the $\phi_{1,2}$-gradients must satisfy $\phi_{1,x}=-\phi_{2,y}$, $\phi_{1,y}=\phi_{2,x}$ (from linearized Ohm's law and the requirement that the disturbance fields vanish at $y=+\infty$). This solution is directly applicable to the wavy-wall problem, for which the drag is presented: for $m=1$, the finite R_m theory gives about the same result as before [loc. cit.] for $\sigma=\infty$, if $R_m>5$.

For $\lambda(m^2+1)/4\pi R_m\ll 1$, expansion of τ in (3) gives for $y\gtrless 0$ the damped wave $\xi_\lambda^{\pm}=A_\lambda e^{i\lambda(x-my)}\ e^{\mp\lambda^2\gamma y}$, $\gamma=m(m^2+1)/8\pi R_m$. This approximate form of (3), valid for sufficiently small λ, can now be used to construct a solution by superposition $\xi(x, y^+)=\int_{-\infty}^{\infty}\xi_\lambda^+ d\lambda$ provided $A^+(\lambda)$, the Fourier transform of $\xi(x, 0^+)$, approaches zero fast enough for $|\lambda|\to\infty$. The analogous Fourier transforms of $\Omega(x, 0)$ and of the rotational parts of the four field components along $y=0^{\pm}$ yield relations between these quantities and their x-derivatives. The boundary conditions can now be set up for a thin-airfoil segment (slope $Y'(x)$ between $\pm c/2L$), and the rotational parts eliminated. The resulting boundary condition for ϕ_1 involves $\phi_{1,x}$, $\phi_{1,y}$, Y', Y'', and also $\phi_{1,xy}$; it may be reformulated as an integrodifferential equation for $\phi_{1,y}$ (including the Kutta condition), but more than the complete formulation is at present not available. Solution by a Glauert series is also discussed and does not seem too promising even in the indirect problem of given chordwise lift distribution.

The remainder of the paper discusses, for a simple pressure distribution, the current penetration into the fluid (perhaps unexpectedly large) and the validity of the assumption $A(\lambda)\to 0$ rapidly for large λ.

G. Kuerti (Cleveland, Ohio)

11759:

Chopra, K. P. Hydromagnetic flow in a circular pipe. Z. Physik **162** (1961), 46–52.

The steady flow of an electrically conducting incompressible fluid in a circular pipe with a uniform magnetic field applied transverse to the direction of flow is considered. It is found that for this geometry also the effect of increasing the magnetic field strength is to flatten the velocity profile. Thus, the classical parabolic profile reduces to a uniform flow in the interior of the cylinder in the limit of strong magnetic fields in much the same fashion as was observed by Hartman and Shercliff.

The direction of flow is taken to be the positive z-axis which is also the axis of the cylinder. The one component of velocity v_z is retained and it is further assumed that v_z is a function only of r. The analysis appears to be approximate in that only the applied field is considered. Indeed it was difficult at times for this reader to follow the author's intent, for example, in the matter of the boundary condition at the wall for the purpose of computing the induced magnetic field. In particular, it was confusing in the manner in which the author was able to assume that v_z was only a function of r, apparently neglect the electric field in the analysis, derive an electric current density which is not solenoidal and still manage to compute a consistent induced magnetic field.

R. R. Gold (El Segundo, Calif.)

11760:

Gogosov, V. V. The motion of a piston in a conducting medium. Dokl. Akad. Nauk SSSR **135** (1960), 30–32 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1160–1162.

Le problème du piston dans un fluide conducteur, en présence d'un champ magnétique a été étudié par de nombreux auteurs. Dans le présent travail, le fluide et le piston ont une résistivité électrique négligeable; le champ magnétique est perpendiculaire au plan du piston et possède une intensité arbitraire. Tous les cas possibles avec ces hypothèses sont discutés. *H. Cabannes* (Paris)

11761:

Linhart, J. G. Dynamic stability of a conducting, cylindrical shell in a magnetic field. J. Appl. Phys. **32** (1961), 500–505.

In this paper the dynamic stability of a thin perfectly conducting shell in axial and azimuthal magnetic fields is investigated. It is assumed that the cylindrical surface is

perturbed by a small amplitude ripple and the corresponding perturbed magnetic field is then obtained. The equation of motion of the conducting surface is then derived and from this the stability criteria are obtained. The analysis is carried out for both internal and external magnetic fields and for the three cases of sausage, flute-like and helical perturbations of the shell.

W. E. Williams (Liverpool)

11762:

Ludford, G. S. S.; Murray, J. D. On the flow of a conducting fluid past a magnetized sphere. J. Fluid Mech. **7** (1960), 516–528.

For incompressible, inviscid fluid of finite conductivity, the non-dimensional equations for the magnetic and mechanical field quantities outside the sphere of rad. a are

$$\begin{aligned} &\text{(a) } \operatorname{div}\mathbf{v} = 0,\\ &\text{(b) } (\operatorname{curl}\mathbf{v})_\Lambda\mathbf{v} = -\operatorname{grad}P+\beta\,(\operatorname{curl}\mathbf{H})_\Lambda\mathbf{H},\\ &\text{(c) } \operatorname{curl}\mathbf{H} = R_M\mathbf{v}_\Lambda\mathbf{H},\\ &\text{(d) } \operatorname{div}\mathbf{H} = 0, \end{aligned} \tag{1}$$

with $P=p+\frac{1}{2}v^2$, $R_M=Ua\mu\sigma$ (magnetic Reynolds number), $\beta=\mu h^2/\rho_0U^2$ (representative ratio of magnetic mechanical energy densities). $\mathbf{E}=0$ because of the assumed axial symmetry. The small parameter is β, the required expansions have the form $v=v_0+\beta v_1+\cdots$, $H=H_0+\beta H_1+\cdots$, $p=p_0+\beta p_1+\cdots$, and the perturbations must satisfy

$$\begin{aligned} &\text{(a) } \operatorname{curl}\mathbf{H}_0 = R_M\mathbf{v}_0{}_\Lambda\mathbf{H}_0,\\ &\text{(b) } \operatorname{div}\mathbf{H}_0 = 0,\\ &\text{(c) } (\operatorname{curl}\mathbf{v}_1)_\Lambda\mathbf{v}_0 = -\operatorname{grad}P_1+(\operatorname{curl}\mathbf{H}_0)_\Lambda\mathbf{H}_0,\\ &\text{(d) } \operatorname{div}\mathbf{v}_1 = 0, \end{aligned} \tag{2}$$

where $P_1=p_1+\mathbf{v}_0\cdot\mathbf{v}_1$. This arrangement of the perturbation expansions makes β and h in a certain respect formally independent: $\mathbf{H}$ need not vanish if $\beta=0$, and the field-free case is only obtained from $\mathbf{H}_{\text{diff}}=\mathbf{H}_0\,h$ by making $h=0$. As a consequence, (2a) and (2b) determine $\mathbf{H}_0$ for the given field-free flow $\mathbf{v}_0$, and (2c), (2d) can then be used to determine $\mathbf{v}_1$ and p_1.

$\mathbf{H}_0$ depends on the solution of

$$A_{rr}+r^{-2}A_{\vartheta\vartheta}-r^{-2}\cot\vartheta A_\vartheta = R_M[(1-r^{-3})\cos\vartheta A_r-r^{-1}(1+\tfrac{1}{2}r^{-3})\sin\vartheta A_\vartheta], \tag{3}$$

where $\{H_{0r}, H_{0\vartheta}\}=\{r^{-1}A_\vartheta, -A_r\}/(r\sin\vartheta)$. For small R_M, a regular perturbation solution is modeled after the product solutions of (3), which hold, if terms r^{-3} and r^{-4} on the right-hand side are omitted, after the factor $\exp[(R_M/2)r\cos\vartheta]$ has been split off. If the field inside the sphere has dipole character (i.e., is a dipole along the symmetry axis for equal permeabilities of sphere (μ') and fluid (μ) if $\mathbf{v}=0$), a linear combination with two arbitrary constants of the solutions

$$A_n = \exp[-(\rho/2)(1-\cos\vartheta)] \times \rho^{1/2}e^{\rho/2}K_{n+1/2}(\rho/2)\sin^2\vartheta P_n'(\cos\vartheta), \quad \rho = R_Mr \tag{4}$$

for $n=1$ and $n=2$ includes terms $O(R_M{}^0)$ and $O(R_M{}^1)$ for which the complete equation (3) gives only corrections $O(R_M{}^2)$; at the same time, the boundary conditions on the sphere can be satisfied up to $O(R_M)$ if the inner field is written as a superposition of the original dipole, the usual uniform field and the lowest spherical harmonic symmetric about the axis (but not about the plane $\vartheta=\pi/2$). All four constants are then determined in terms of μ' and μ. The dipole strength defines h.

The vorticity ω ($=\operatorname{curl}\mathbf{v}_1$) is determined by (2c) after elimination of P_1. The resulting first-order equation (5) admits integration along the streamlines of the field $\mathbf{v}_0$:

$$\omega = r\sin\vartheta\int_{-\infty}^{\phi_0 r} r^5[(r^3-1)^2\cos^2\vartheta+(r^3+\tfrac{1}{2})^2\sin^2\vartheta]^{-1} \times f(r,\vartheta;R_M)\,d\phi_0. \tag{6}$$

In the integrand, $f(r,\theta;R_M)=(r\sin\vartheta)^{-1}[\partial(rF_\phi)/\partial r-\partial F_r/\partial\vartheta]$, with $\{F_r, F_\phi\}=(\operatorname{curl}\mathbf{H}_0)_\Lambda\mathbf{H}_0$, and r and ϑ must be expressed by ϕ_0 and ψ_0, ψ_{0P} being kept constant. The wake factor (similar to Oseen's approximation in viscous flow) is here $\exp[-R_Mr(1-\cos\vartheta)]$; thus its size is half that of the magnetic wake [cf. (4)]. From (6) it is shown that ω becomes weakly infinite on the sphere and only there: $\omega=-\frac{1}{4}\sin\vartheta_P f(1,\pi;R_M)\log(r_P-1)+$ regular terms, if $r_P\to1$. Another method for ω uses perturbation series. In the irrotational case, $\psi=(A_nr^{n+1}+B_nr^{-n})\sin^2\vartheta P_n'(\cos\vartheta)$, the solution of

$$\psi_{rr}+r^{-2}\psi_{\vartheta\vartheta}-\cot\vartheta\psi_\vartheta = -r\sin\vartheta\omega \quad\text{with } \omega=0. \tag{7}$$

Thus ω should be written as a sum of $\omega_n(r)\sin\vartheta P_n'(\cos\vartheta)$, and $\omega_n(r)$ determined from (5), whose right-hand side, $f(r,\vartheta;R_M)$, must therefore be written as

$$\sum_1^\infty\Omega_n(r)P_n'(\cos\vartheta).$$

The resulting recurrence relation between ω_{n+1}', ω_{n+1}, ω_{n-1}', ω_{n-1} and Ω_n gives, for $n=1$,

$$\omega_3 = [5r^3/3(r^3-1)^2]\int_1^r(\alpha^3-1)\Omega_1(\alpha)\,d\alpha;$$

$\omega_4, \omega_6, \cdots$ can be determined in succession (the odd ω_n depend on the choice of ω_1). ψ_1 (stream function of $\mathbf{v}_1$) is now expanded as $\sum_1^\infty\psi_{1n}(r)\sin^2\vartheta P_n'(\cos\vartheta)$, and ψ_{1n} satisfy $\psi_{1n}''-[n(n+1)/r^2]\psi_{1n}=-r\omega_n$. This gives $v_{1\vartheta}$ and v_{1r} by differentiation.

The pressure drag follows by integration. For the dimensionless result,

$$(\text{drag force}/\rho_0U^2a^2) = D_p = (4/3)\pi\int_1^\infty[a_1(\alpha)+b_1(\alpha)]\alpha^{-1}d\alpha,$$

where $a_n(r)$ and $b_n(r)$ are expansion coefficients of F_ϕ and F_r in $\sin\vartheta P_n'$ and P_n' respectively. For the dipole field, $D_p=\beta O(R_M{}^3)$. The magnetic drag follows by integration of the Maxwell stresses: $D_M=\beta[(8/5)\pi\{3\mu'/(2\mu+\mu')\}^2R_M+O(R_M{}^2)]$.

This analysis does not hold for infinite conductivity of the sphere material, but can be easily adapted to that case, the results being given in the paper.

G. Kuerti (Cleveland, Ohio)

11763:

Bazer, J.; Fleischman, O. Propagation of weak hydromagnetic discontinuities. Phys. Fluids **2** (1959), 366–378.

Les auteurs étudient la propagation des perturbations infiniment petites dans un fluide compressible dont le résistivité électrique est négligeable. Suivant le modèle de l'optique géométrique les fronts d'ondes sont construits à partir des rayons (normales aux surfaces d'ondes). Les formules qui déterminent la variation de l'intensité des discontinuités le long des rayons sont établies. Les résultats sont appliqués à la solution du problème du piston

dans le cas où le résistivité électrique du piston est également négligeable et où le champ magnétique, supposé initialement constant, est incliné sur le normale au piston.

H. Cabannes (Paris)

11764:

Hillion, Pierre; Takabayasi, Takehiko; Vigier, Jean-Pierre. Relativistic hydrodynamics of rotating fluid masses moving with the velocity of light. Acta Phys. Polon. **19** (1960), 245–270.

F. Halbwachs, P. Hillion and J.-P. Vigier have previously [Nuovo Cimento (10) **10** (1958), 817–833; MR **21** #1188] given a Lagrangian formulation of the equations of motion of relativistic fluid masses. In this paper a limiting form of this Lagrangian is obtained which describes such motions when the velocity of the center of matter density approaches the velocity of light. It is further shown that the tensor equations equivalent to the neutrino equations may be derived from a Lagrangian similar to the one obtained but differing from it by the presence of certain terms interpreted by the authors as contributing "tensions" to the fluid.

A. H. Taub (Urbana, Ill.)

11765:

Skobelkin, V. I. Variational principles in hydrodynamics. Soviet Physics. JETP **4** (1957), 68–73.

The principle of the least streaming potential is defined and it is generalized to the relativistic flow for a stationary field. The principle of least action for a non-stationary field is set up, and the existence of a strong minimum for the streaming potential is established.

G. C. McVittie (Urbana, Ill.)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also A10920, 11552, 11557, 11742, 11757, 11955, 12001.

11766:

Moon, Parry; Spencer, Domina Eberle. ★Field theory for engineers. The Van Nostrand Series in Electronics and Communications. D. Van Nostrand Co., Inc., Princeton, N.J.-Toronto-London-New York, 1961. ix+530 pp. $12.75.

Field theory is a subject which has long been interesting to engineers, especially to electrical engineers, and of late that interest, as well as that for mathematical methods, has been accelerated since the war. Inadequate mathematical skill is often felt to be an impediment, and the aim of this book is to remedy this deficiency. On the other hand, it is important that mathematicians, at least some of them, should have a feeling for engineering problems, and here, also, the book may render a good service. It is somewhat dubious as to what kind of preliminary knowledge the authors expect from the reader. They explain carefully the notations used in the system of coordinates, but in other instances they seem to suppose the reader to be familiar with several mathematical theorems.

The authors give a list of subjects covering a wide range of application of field theory. They distinguish scalar fields and vector fields and, in view of classification of the latter, they introduce the divergence and curl operators. In the first place they treat scalar fields, especially those that can be described with the aid of Laplace's equation and allied equations, Poisson's equation, the diffusion equation and the wave equation. They present these equations in diffferent systems of coordinates, limiting themselves generally to orthogonal coordinates, and discuss methods of constructing solutions, particularly for Laplace's equation. The greater part of the book is devoted to the method of separation of variables, and a chapter deals with the conditions to be imposed on the coordinate-system to assure separability. A considerable number of examples is given to show how, with the aid of the solutions obtained by this method, the solution can be constructed that satisfies given boundary conditions. Dirichlet conditions, Neumann conditions and mixed conditions are considered. Here of course Fourier's theorem comes into action. The examples show also how the character of the boundary conditions suggests the system of coordinates to be introduced. The solutions of the differential equations obtained by separation of variables involve, in addition to trigonometric and exponential functions, Bessel, Legendre, Baer, Laguerre, Hermite, Mathieu and Lamé functions. Laplace's equation is given in rectangular, circular-cylinder, elliptic-cylinder, parabolic-cylinder, spherical, spheroidal and parabolic coordinates. Besides scalar potential, the quasi-potential and the vector potential are discussed.

To each chapter is added a rather extensive list of references and a number of problems. The mathematician will miss existence theorems and a general discussion of the uniqueness of the solution under various boundary conditions. However, for Laplace's equation, some uniqueness theorems are treated in the chapter on rectangular coordinates. Moreover, the numerous examples may throw light on questions of this kind.

H. Bremekamp (Delft)

11767:

Snitzer, E. Cylindrical dielectric waveguide modes. J. Opt. Soc. Amer. **51** (1961), 491–498.

If light is propagated down a circular dielectric cylinder by total internal reflection, and if the diameter of the cylinder is comparable to the wave length of the light, then only certain field distributions, or modes, will satisfy Maxwell's equations and the boundary conditions. In this paper it is shown that there are two sets of solutions; the solutions are obtained and their properties discussed, including limiting cases far from cutoff and at cutoff (corresponding to the critical angle of incidence in geometrical optics).

G. L. Walker (Providence, R. I.)

11768:

Györgyi, G. Elementary considerations on the dynamics of light waves. Amer. J. Phys. **28** (1960), 85–88.

A discussion of light refraction and Čerenkov radiation is given, based on dynamical considerations. It is shown that the effects, sometimes described by means of the Minkowski energy momentum tensor, can also be described by means of the energy momentum tensor of Abraham. It is also argued that the Abraham tensor, but not that of Minkowski, yield the correct expression for the Lorentz force acting on polarization currents.

E. Wolf (Rochester, N.Y.)

11769:

Raman, C. V.; Pancharatnam, S. The optics of mirages.

Proc. Indian Acad. Sci. Sect. A **49** (1959), 251–261. (1 plate)

It is pointed out that the explanations usually given (on the basis of geometrical optics) of the formation of mirages are unsatisfactory. The authors investigate the problem by means of wave theory and obtain results in good agreement with observations.

E. Wolf (Rochester, N.Y.)

11770:

Graff, G.; Jánossy, L. An investigation concerning the classical fluctuation of light. Acta Phys. Acad. Sci. Hungar. **10** (1959), 291–300. (Russian summary)

Formulae are derived relating to classical fluctuations of a light beam. The results are applied to determine the coincidence rate in the output from two photomultipliers illuminated by two beams which are derived from a single beam by means of a semi-transparent mirror.

E. Wolf (Rochester, N.Y.)

11771:

Barakat, Richard. Total illumination in a diffraction image containing spherical aberration. J. Opt. Soc. Amer. **51** (1961), 152–157.

Author's summary: "The total illumination (or encircled energy) in a diffraction image containing spherical aberration is studied for various amounts of third- and fifth-order aberration. The Zernike polynomials are used to represent spherical aberration. The evaluation of the total illumination was done numerically, using double Gauss quadrature and 49 quadrature points. Contour maps of the total illumination were constructed and compared with the aberration-free case examined previously by E. Wolf."

11772:

Czerny, M. Ein Verfahren zum vereinfachten Rechnen mit dem Planckschen Strahlungsgesetz. Optik **17** (1960), 671–677. (English and French summaries)

A graphical method is given for rapid evaluation of certain integrals arising in optical problems involving passage of light through filters.

E. W. Marchand (Rochester, N.Y.)

11773:

Neugebauer, Th. Über die Interferenz von Photonen. Acta Phys. Acad. Sci. Hungar. **12**, 241–247 (1960). (Russian summary)

The author seeks an explanation of the fact that different low-frequency photons can interfere with each other, while apparently at the frequency of visible light and higher only the interference of a photon with itself is observed. Using approximate classical and quantum-mechanical formulas for energy levels of idealized emitters he proposes the explanation that successive photons from the usual sources at radio frequencies are in phase or nearly in phase, making mutual interference possible. If similar phase-continuous sources at visible wave-lengths and shorter were available, mutual interference would be observable here also. He does not mention the optical maser as such a possible source. *O. Frink* (Dublin)

11774:

Rowe, J. E.; Meeker, J. G. Interaction of premodulated electron streams with propagating circuits. J. Electronics Control (1) **9** (1960), 439–466.

Linear and nonlinear analyses are presented for prebunched beams interacting with an r.f. wave in growing-wave and beating-wave devices. The increase in gain in the linear case is calculated, and conditions are developed analytically for optimizing the increase with respect to predrift angle for the stream and initial phase of the r.f. wave. The increase in efficiency and gain and the decrease in length for optimally bunched beams are calculated in the nonlinear case, and the process of phase focusing a nearly ideal bunch in the presence of a circuit field and space of charge forces is analyzed.

J. E. Rosenthal (Passiac, N.J.)

11775:

Bevensee, Robert M. A unified theory of electron beam interaction with slow wave structures, with application to cut-off conditions. J. Electronics Control (1) **9** (1960), 401–437.

An analysis is given of the interaction between an electron beam and a general slow-wave structure representative of the lightly coupled cavity chain. The fields are expanded in sets of solenoidal and irrotational modes of an empty cavity bounded by either electric or magnetic shorting planes on the coupling surfaces. Small-signal transmission-line equations including relativistic effects are obtained for a longitudinally confined beam under the action of cavity solenoidal electric field and space irrotational field. The analysis is applied to a study of the beam-circuit interaction when the slow space charge beam mode is nearly synchronous with the "cold" circuit near the latter's cut-off. *J. E. Rosenthal* (Passaic, N.J.)

11776:

Edwards, S. F. The charge correlation function of a plasma in a magnetic field. Philos. Mag. (8) **6** (1961), 61–69.

Calling $\rho(\mathbf{x})$ the density fluctuation in a plasma, defined as $\rho(\mathbf{x})=\sum_a \delta(\mathbf{x}-\mathbf{x}_a)-N/V$ ($\mathbf{x}_a$ being the positions of the particles, N their number and V the volume), the central quantity studied in this paper is the correlation function in equilibrium: $\langle \rho(\mathbf{x}, t)\rho(\mathbf{x}', t')\rangle$. An integral equation is set up for this quantity by making a Brownian motion type of assumption, i.e., assuming that the paths of the particles deviate but slightly from a straight line. The equation can be solved in terms of a Laplace transform both in the absence and in the presence of an external magnetic field. Especially simple expressions are obtained for the time integral of this function, which is involved in the theory of transport coefficients.

R. Balescu (Brussels)

11777:

Edwards, S. F.; Sanderson, J. J. A new approach to transport problems in fully ionized plasmas. Philos. Mag. (8) **6** (1961), 71–87.

An expression for the electrical conductivity is given in terms of one- and two-body Green's functions of a many-particle problem in classical statistics. Approximate equations of evolution are derived for these functions by using a kind of superposition approximation for the high-order correlation functions. The validity of this approximation is discussed. The theory is illustrated by the simple case of a Lorentz gas (independent electrons and heavy fixed ions).

It is shown that in this case the inclusion of dynamical collective screening effects does not modify the results obtained by traditional methods (Fokker-Planck equation with a Debye potential).

No mention is made of R. Kubo's work [J. Phys. Soc. Japan **12** (1957), 570–586; MR **20** #4940a] in which an equivalent expression for the electrical conductivity is obtained (although no methods are given there for the evaluation of the correlation functions).

R. Balescu (Brussels)

11778:

Colgate, Stirling A.; Furth, Harold P. Stabilization of pinch discharges. Phys. Fluids **3** (1960), 982–1000.

The paper gives an extensive and illuminating study of the stability of a hard-core pinch. The experiments conducted show that such pinches are more stable than the conventional linear pinch with respect to both hydromagnetic and other instabilities. From the analysis of the equation of motion, it has been shown that the plasma current in the direction perpendicular to the magnetic field is responsible for plasma heating, while the current in the direction of magnetic field is a disporable parameter. Also from the equation describing the change in the magnetic field with time, the rate of change of magnetic energy has been shown to be negative and is linearly proportional to plasma resistivity, the latter being proportional to $T^{-3/2}$, where T is the temperature. Hence, as the temperature increases, this diffusion heating is found inadequate to counterbalance the energy loss due to ionization, radiation, etc., so that the slow diffusion of the magnetic field is necessary for stable configurations and to confine plasmas.

The large Doppler broadening and the neutron emission from cold deuterium plasma experiments show the existence of small-scale instabilities. Also the plasma resistivity has been found to be much larger than the theoretical value above a certain critical temperature, due essentially to interaction between the cooperative phenomena and the electron beam. Thus these small-scale instabilities and the hydromagnetic instabilities should be removed for plasma confinement for considerable duration. The presence of finite surface current introduces gross instability in quasi-static linear pinch, thereby making it impossible to test the validity of any hydromagnetic stability theory experimentally. But the hard-core pinch, which affords a current conduction through the central core, is found to be more stable with regard to hydromagnetic stability, provided a toroidal magnetic field B_θ can be produced inside the plasma, which satisfies Suydam's necessary condition for hydromagnetic stability, namely,

$$\frac{dB_\theta/dr}{B_\theta} < -\frac{1}{r}$$

Such a field is possible to realize in the present set-up of the experiments. However, even when this criterion is fulfilled, there are certain non-hydromagnetic instabilities present. A rapid rise of pinch current and introduction of more air in the deuterium plasma increases the stability. It has also been shown that the curvature of the magnetic field lines due to inside and outside pinch and the process by which the external pinch is produced do not make any marked difference in the instability.

Local disturbances of magnetic and electric fields may produce a quasi-steady turbulence owing to dissipation of current density due to the finite resistivity to these disturbances. But a sufficiently high magnetic field is impervious to such instabilities, provided the outer pinching mechanism has high intensity.

The density and distribution of ions are studied by introducing sharp pulses or modulations in the outer pinch field and observing the mean transit velocity of the compressional waves produced. In spite of assuming uniform magnetic field inside the plasma, it has been observed to agree well with experimental results.

The authors also discuss the effects of electrodes on the stabilized hard-core pinches. The presence of electrodes introduces asymmetry effects, but the reversal of the direction of the axial magnetic field does not affect the stability. Finally, the prospects of stable high-temperature operation in a Levitron are discussed.

P. L. Bhatnagar (Bangalore)

11779:

Scheuer, P. A. G. The absorption coefficient of a plasma at radio frequencies. Monthly Not. Roy. Astr. Soc. **120** (1960), 231–241.

This paper offers a useful re-examination of the classical collision processes that affect radio-frequency radiation in a plasma. The theory encounters difficulty owing to the fact that the majority of encounters between electrons and ions is not binary. Smerd and Westfold [Philos. Mag. **40** (1949), 831–848] calculated the contributions from all binary encounters for which the collision parameter did not exceed the mean interionic distance b_1, and neglected the remaining, essentially multiple, encounters. On the other hand there was Elwert's [Z. Naturforschg. **3A** (1948), 477–481] quantum-mechanical formula which, though concerned with binary encounters, was obtained without the notion of collision parameters. It corresponded to the classical formula with a limiting collision parameter equal to the distance b_2 travelled by an electron within a period of the radiation. Whether the contributions from multiple encounters are indeed negligible was open to question. Later work by Cohen, Spitzer, and Routly [Phys. Rev. (2) **80** (1950), 230–238; MR **13**, 307] suggested that their effect might be accounted for by treating all collisions as binary with the Debye length b_3 replacing b_1 as the limiting collision parameter.

The present attack consists in summing the contributions from distant encounters $b > b_0$, calculated for an undeviated path through the field due to a random distribution of ions, and the contributions from close encounters $b < b_0$ calculated for binary collisions. The critical value b_0 is chosen so that $b_1 \gg b_0 \gg b_4$, where b_4 is the collision parameter corresponding to a deflection of $\pi/2$. The total sum is independent of b_0 and, indeed, agrees with Elwert's formula. The neglect of Debye shielding is justified. {It should be mentioned that the refractive index should occur in the numerators instead of the denominators of formulas (1), (4), (5), (6) for the emitted power.}

K. C. Westfold (Clayton)

11780a:

Kalman, G. Nonlinear oscillations and nonstationary flow in a zero temperature plasma. I. Initial and boundary value problems. Ann. Physics **10** (1960), 1–28.

11780b:

Kalman, G. Nonlinear oscillations and nonstationary flow in a zero temperature plasma. II. General characteristics of the motion. Ann. Physics **10** (1960), 29–61.

The author shows that the equations for the nonlinear dynamics of a one-dimensional cold-electron plasma become linear in terms of Lagrangian coordinates, apparently unaware that this has already been observed and utilized by John M. Dawson, Phys. Rev. (2) **113** (1959), 383–387 [MR **21** #560]. He gives a number of interesting applications to particular cases, and discusses energy partitions and spectral decompositions.

M. Kruskal (Princeton, N.J.)

11781:

Volkov, T. F. Stability of a plasma cylinder in a high-frequency magnetic field. Ž. Tehn. Fiz. **30** (1960), 497–503 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 467–472.

The stabilizing of a plasma configuration by means of a quasistationary high-frequency magnetic field is considered (by "high-frequency" one means that the frequency associated with particles moving at thermal velocities over distances of the order of the system's dimensions is much less than the frequency of the magnetic field). The problem is solved in the hydrodynamic approximation. It is found that such an external field, of high enough amplitude and travelling along an axis, can stabilize a plasma cylinder with respect to arbitrary small perturbations. A field which rotates azimuthally does not provide stability. *R. M. May* (Cambridge, Mass.)

11782:

Larkin, A. I. Thermodynamic functions of a low-temperature plasma. Ž. Éksper. Teoret. Fiz. **38** (1960), 1896–1898 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1363–1364.

Author's summary: "We have evaluated the thermodynamic functions of a plasma at temperatures below the ionization temperature. If the interaction of the ions with the electrons of the continuous spectrum is taken into account properly, it will partially compensate the contribution from the excited states of the atoms and complex ions." *R. M. May* (Cambridge, Mass.)

11783:

Rosenbluth, Marshall N. Long-wavelength beam instability. Phys. Fluids **3** (1960), 932–936.

This paper is devoted to a study of the long-wavelength "kink" instability of a neutralized self-pinched beam of particles, immersed in a resistive plasma. (This beam is described by a relativistic Boltzmann equation; the plasma, by a simple Ohm's law with low conductivity.) First it is shown that, for oscillations whose frequency is small enough, the beam is able to displace itself in a manner which is essentially rigid. The background plasma then provides a viscous drag upon such a displacement—this leads to an instability of the beam.

R. M. May (Cambridge, Mass.)

11784:

Linhart, J. G. Theory of fusion reactions in an unconfined plasma. Nuovo Cimento (10) **17** (1960), 850–863. (Italian summary)

A criterion for a zero-energy gain-fusion reactor is obtained, the reactor resembling a cylindrical H-bomb in which a plasma is either not confined or only marginally confined. A magnetically driven cylindrical plasma shell is shown to generate a plasma column for which the above mentioned criterion could be satisfied.

Glen J. Morris (Seattle, Wash.)

11785:

Barabanenkov, Yu. N. On the solution of the kinetic equation for a plasma in a variable magnetic field. Ž. Éksper. Teoret. Fiz. **37** (1959), 427–429 (Russian); translated as Soviet Physics JETP **10** (1960), 305–306.

Author's summary: "The motion of a completely ionized plasma (collisions being neglected) along a narrow magnetic tube of an axially symmetrical magnetic field is considered by means of the kinetic equation. The equation is solved under the assumption of sufficiently slow variation of the magnetic field. Canonical variables are chosen as the independent variables of the distribution function."

H. C. Kranzer (Garden City, N.Y.)

11786:

Cerkovnikov, J. A. Die Konvektions-Instabilität eines verdünnten Plasmas. Fortschr. Physik **8** (1960), 528–548.

Certain models for inhomogeneous equilibrium configurations of a plasma are tested for stability against local infinitesimal disturbances of very short wavelength. The initial disturbance is assumed to have space dependence $\exp(i\mathbf{k}\cdot\mathbf{x})$ where, ϕ being any equilibrium field quantity, $\mu=\mathbf{k}\cdot\nabla \ln\phi \ll 1$. In the linearized perturbation equations, only the lowest order terms in μ are retained. A necessary condition for local stability is given for a perfectly conducting ideal gas in a curved magnetic field or a straight magnetic field and gravitational field. Stability criteria are given for a two-fluid model of a plasma with electron drift in a straight magnetic field without gravity; for a heat conducting, collisionless plasma in which the heat flow is computed by assuming that the perturbed Boltzmann distributions are Maxwellian; and for a collisionless plasma whose deviation from an initial Maxwell distribution is calculated by means of the Boltzmann equation.

G. E. Backus (La Jolla, Calif.)

11787:

Scarf, F. L. Wave propagation in a moving plasma. Amer. J. Phys. **29** (1961), 101–107.

This paper examines the electromagnetic properties of a moving plasma, the relativistic transformation laws being given for the index of refraction. The Fresnel drag is then determined for various values of the temperature, collision frequency, natural frequency and external magnetic field. The emphasis is on the physical explanation of the existence and magnitude of the drag. Simple models are used and ionic motion is neglected. The author concludes that his analysis suggests that careful measurements of the Fresnel drag, as a function of the electromagnetic frequency, could give information about the state of the medium. *J. A. Morrison* (Murray Hill, N.J.)

11788:

Vandakurov, Yu. V. Stability of a thin circular plasma conductor in a magnetic field. Ž. Tehn. Fiz. **30** (1960),

330–337 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 305–312.

The stability of a toroidal pinch with major radius of curvature large compared to the minor radius of a surrounding perfectly conducting toroidal wall is ordinarily treated as if it were an infinite cylindrical pinch, but with the longitudinal wavelengths of the perturbations restricted to lengths fitting an integral number of times into the original major circumference. In this paper the validity of the described procedure is verified by writing out in detail the equations for neutral normal modes ($\omega = 0$), which give the boundaries of regions of stability (by the principle of exchange of stability, which is known to be valid in this type of problem), and checking that the boundaries agree (to be lowest order in an expansion in the ratio of radii) for the two treatments.

M. Kruskal (Princeton, N.J.)

11789:

Rostoker, Norman. Kinetic equation with a constant magnetic field. Phys. Fluids **3** (1960), 922–927.

In a previous paper [N. Rostoker and M. N. Rosenbluth, Phys. Fluids **3** (1960), 1–14; MR **22** #4340] an expansion method was given to solve the Liouville-hierarchy for the singlet and pair distribution functions in a completely ionized gas. The expansion parameter is essentially the reciprocal number of particles within a Debye-sphere. In this paper this method is applied to derive a non-linear integro-differential equation, the Fokker-Planck equation for the singlet distribution function of a spatially homogeneous plasma subject to a constant external magnetic field.

N. L. Balazs (Princeton, N.J.)

11790:

Fried, Burton D.; Gould, Roy W. Longitudinal ion oscillations in a hot plasma. Phys. Fluids **4** (1961), 139–147.

Linearized, longitudinal waves in a hot plasma include, besides the familiar electron plasma oscillations in which the frequency ω is of order $\omega_p = (4\pi ne^2/m)^{1/2}$, also ion plasma oscillations with $\omega \approx \omega_p(m/M)^{1/2}$. The properties of the latter are explored using a Vlasov-equation description of the plasma. For equal ion and electron temperatures, $T_e = T_i$, there exists a discrete sequence of ion oscillations, but all are strongly damped, i.e., have $-\mathrm{Im}\,\omega/\mathrm{Re}\,\omega \gtrsim 0.5$, and hence are not likely to be observable. The ratio $\mathrm{Im}\,\omega/\mathrm{Re}\,\omega$ can be made to approach zero (facilitating detection of the waves) by either increasing T_e/T_i or by producing a current flow in the plasma. In the latter case, $\mathrm{Im}\,\omega$ can even be made positive (corresponding to growing waves), the current required for this being smaller the larger the value of T_e/T_i. This growing wave is just the familiar two-stream instability which is thus seen to be an unstable ion oscillation. It is also noteworthy that the ion oscillations, which for small k have the properties usually associated with an acoustic wave (longitudinal polarization, $\omega \propto k$), are obtained using a formalism which is sometimes designated as "collisionless".

N. L. Balazs (Princeton, N.J.)

11791:

Weibel, Erich S. Dynamic stabilization of a plasma column. Phys. Fluids **3** (1960), 946–960.

Author's summary: "A cylindrical plasma column is confined by a magnetic field whose z-component is constant while the ϕ-component oscillates sinusoidally. The skin depth is assumed to be negligible so that the boundary can be considered as sharp and the interior of the plasma as field-free. The plasma itself is treated as an assembly of noncolliding particles which are specularly reflected at the plasma surface. The oscillating pressure of the applied field causes the plasma surface to execute a small periodic motion independent of ϕ and z. All other components of the displacement decay in time. Thus the plasma is positively stable against all deformations. For magnetostatic confinement this treatment yields the usual instabilities, but, in contrast to earlier work, the growth-rates are bounded as the deformation wavelength approaches zero. This appears to be in better agreement with experiment."

H. C. Kranzer (Garden City, N.Y.)

11792:

Poincelot, Paul. Généralisation de la condition aux arêtes. C. R. Acad. Sci. Paris **250** (1960), 4316–4318.

The author investigates the admissible singularities of field vectors, currents and charges at the edge of a solid perfect conductor. However, his results have been known for a long time [A. W. Maue, Z. Physik **126** (1949), 601–618; MR **11**, 293].

C. J. Bouwkamp (Eindhoven)

11793:

Van Bladel, J. On Helmholtz's theorem in multiply-bounded and multiply-connected regions. J. Franklin Inst. **269** (1960), 445–462.

From the author's summary: "The splitting of a continuous vector function into a gradient plus a curl is possible in an infinite number of ways. The present paper is basically concerned with two particular uniquely determined splittings called electric and magnetic. Eigenvector expansions suitable for these splittings are examined, and the application of these expansions to the calculation of the various terms of the splitting is considered."

H. D. Block (Ithaca, N.Y.)

11794:

Ford, Fred. The calculation of the electromagnetic field in an Alvarez type linear accelerator. Proc. Cambridge Philos. Soc. **56** (1960), 252–268.

This paper is concerned with the calculation of the electromagnetic field in a linear accelerator. A recent work by Wild [Quart. J. Mech. Appl. Math. **10** (1957), 322–341; MR **19**, 496] concerning the calculation of electromagnetic fields in nearly-periodic structures is extended to provide a practicable method for computation for resonant structures, particularly for structures of the type used by Alvarez [Rev. Sci. Instrum. **26** (1955), 111–133]. In a periodic system formed by joining together a number of equal cavities, aperture-to-aperture, there can exist a series of electromagnetic fields called transmission modes. The cavity dimensions in the Alvarez accelerator are determined as functions of position by considerations of particle dynamics and others. The purpose of this paper is to calculate the effect on the acceleration characteristics of a small departure from exact periodicity and of small departures from exact resonance in the cavities.

K. Noda (Tokyo)

11795:
Lindsay, P. A. General steady-state theory of cylindrical magnetrons. J. Electronics Control (1) **9** (1960), 241–283.

This paper develops a general steady-state theory of cylindrical magnetrons, taking into account space-charge and the 3-dimensional Maxwellian distribution of the emission velocities of the electrons. (Previous workers have limited their analysis to the spread of initial velocities in only one direction.) The approach adopted is that of applying Liouville's theorem, which yields the number of electrons in an element of phase-space at an arbitrary point between the electrodes, from the known phase-space density of electrons immediately outside a cylindrical emitter. From the equations of motion, the velocity components at an arbitrary point are expressed in terms of the (unknown) electrostatic potential and the initial velocity at the cathode. The volume density of electrons is obtained by integrating the phase-space density with respect to the velocity components, and, in order to do this, the limits of integration must be determined. This involves a lengthy discussion of electron trajectories.

In the cylindrical magnetron there are three different effects which can balance each other, namely the centrifugal force, the magnetic field and the electric field. Thus there are two possible initial velocities (depending on the various parameters) for which a tangential electron can graze the anode. The first case considered is the simplest one of a temperature-limited magnetron, in which all electrons capable of emission are in fact emitted and are all collected by the anode. The radial and azimuthal components of the current density, as well as the volume density of electrons, are determined. The second case is that of the magnetron which is operated in what is denoted by the "magnetic-field-limited" region. This covers a wide range of operation before the valve becomes space-charge-limited, and is characterized by the predominant role of the magnetic field which is largely responsible for the shape of the electron trajectories, rather than the electric field. There exists a critical potential distribution which separates the magnetic-field-limited and the space-charge-limited regions. It is found that in a well "cut-off" magnetron the azimuthal component of the current density may be several hundred times larger than the radial component. *J. A. Morrison* (Murray Hill, N.J.)

11796:
Landau, L. D.; Lifshitz, E. M. ★Electrodynamics of continuous media. Course of Theoretical Physics, Vol. 8. Translated from the Russian by J. B. Sykes and J. S. Bell. Pergamon Press, Oxford-London-New York-Paris; Addison-Wesley Publishing Co., Inc., Reading, Mass.; 1960. x+417 pp. $12.50.

This is an excellent translation of the Russian original which has been reviewed already [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957; MR **20** #6274]. Compared to the original the following differences are noted: misprints are corrected; printing, paper and binding excellent; price high. *N. L. Balazs* (Princeton, N.J.)

11797:
Fletcher, P. C.; Kittel, C. Considerations on the propagation and generation of magnetostatic waves and spin waves. Phys. Rev. (2) **120** (1960), 2004–2006.

Da nach die neuesten Untersuchungen der ferrimagnetische Stoff Yttrium-Eisen-Granat (Garnet) besonders lange magnetische Relaxationszeiten besitzt, so würde es interessant sein, die Ausbreitung von Spinwellen in einem aus diesen Material hergestellten Stabe experimentell zu untersuchen. Ziel der vorliegenden Arbeit ist die Theorie der zu erwartenden Erscheinungen auszuarbeiten. Besprochen wird die Dispersionsgleichung, die Verteilung der Magnetisierung und die Gruppengeschwindigkeit von magnetostatischen Wellen in einem unendlichen Kreiszylinder mit einem statischen magnetischen Feld (H_0) entlang seiner Achse. Es wird darauf aufmerksam gemacht, dass es ein verhältnismässig grosses Gebiet im Wertbereich des Wellenvektors k gibt, in dem einfache magnetostatische Wellen sich ausbreiten können. Aus einer Grenzbedingung dieser Wellen an der Grenzfläche ($r=R$) des Zylinders folgt dann $F_0(x)=0$, wo F die Besselschen Funktionen bedeutet und x eine Funktion von k, R, γ, H_0 und ω ist. Für die Dispersionsgleichung folgt weiter (wenn die Winkelabhängigkeit gleich $e^{i\nu}$ ist), unter der Annahme dass $kR \gg 1$ ist

$$\omega \simeq \gamma H_0 + \gamma 2\pi M_s (x_i/kR)^2 + (D/\hbar)k^2, \tag{1}$$

wo M_s die Sättigungsmagnetisierung und $\gamma = e/2mc$ das gyromagnetische Verhältnis bedeutet, wo die x_i die Wurzeln von $F_0(x)=0$ sind, und D die Austauschkonstante ist. Ohne dem letzten Gliede bezieht sich (1) auf dem einfachen magnetostatischen Fall, mit diesem dagegen auf Spinwellen.

Die erhaltenen Resultate werden auch graphisch dargestellt und in einem Anhang wird der Fall $H_0=0$ besprochen. *T. Neugebauer* (Budapest)

11798:
Fierz, M. Zur Anziehung leitender Ebenen im Vakuum. Helv. Phys. Acta **33** (1960), 855–858.

11799:
Casimir, H. B. G. A note on multipole radiation. Helv. Phys. Acta **33** (1960), 849–854.

11800:
Williams, W. E. Vertex generated waves outside metallic wedges. Proc. Cambridge Philos. Soc. **57** (1961), 393–400.

Author's summary: "A study is made of the waves generated by a magnetic line source placed at the vertex of a wedge of high conductivity and arbitrary angle. The boundary-value problem is reduced to the solution of a difference equation and an exact solution obtained. The method is also applied to the case of dielectric coated wedges where the surface reactance and resistance are arbitrary, and the propagation of surface waves along such surfaces is considered briefly. The forms of the solution for large and small values of the surface impedance are obtained and show complete agreement with the known results available for a right-angled wedge and a plane."
R. D. Kodis (Providence, R.I.)

11801:
Toraldo di Francia, G. On the theory of some Čerenkovian effects. Nuovo Cimento (10) **16** (1960), 61–77. (Italian summary)

An expansion for the field of a uniformly moving charge is obtained in terms of waves which are exponentially attenuated in a rectilinear direction perpendicular to the direction of motion. This expansion is valid in the half-space extending from the line of motion in the direction of the exponential decrease.

The result is then applied to the calculation of the energy radiated by the charge in the ordinary Cerenkov effect and in the cases where the charge is moving parallel to the surface of a dielectric half-space or a diffraction grating. *E. T. Kornhauser* (Providence, R.I.)

11802:

Schmitz, G.; Uhlenbusch, J. Zur mathematischen Beschreibung zylindersymmetrischer Lichtbögen. Z. Physik **159** (1960), 554–562.

Authors' summary: "Nach Einführung der Wärmestrom-Diffusionsfunktion S wird die Elenbaas-Hellersche Differentialgleichung

$$F(S) = \frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dS}{d\rho}\right)$$

integrierbar gemacht, indem man die Funktion $F(S)$ durch drei Streckenzüge linearisiert. Aus der Lösung $S(\rho)$ kann der radiale Temperaturverlauf im Lichtbogen berechnet werden. Weiter werden Ausdrücke für die elektrisch zugeführte Leistung sowie die abgestrahlte Lichtleistung angegeben."

11803:

Ryffert, Halina. Distribution of the instantaneous waveform of oscillation into almost-orthogonal pulses. Proc. Vibration Problems No. 4 (1960), 37–47. (Polish and Russian summaries)

The author introduces the concept of "almost orthogonality" of a family of periodic functions and pulses, and makes the corresponding generalization of the concept of completeness of an "almost orthogonal" expansion.

V. M. Papadopoulos (Melbourne)

11804:

Low, F. E. Correspondence principle approach to radiation theory. Amer. J. Phys. **29** (1961), 298–299.

In this paper, in the case of the electric dipole radiation, the reduction of the classical formula for the spontaneous radiation by an electron to a form compatible with the quantum-theoretical concept is discussed without making further assumptions. Starting from the diagonal operator for a classical displacement and considering degeneracy and the asymmetry of the frequency, the author finds the number of photons of given frequency radiated per unit time in such a form that there is no interference between states of different energy, and the only possible transitions are downward. *S. Ueno* (Santa Monica, Calif.)

11805:

Ter-Mikaelyan, M. L. Radiation fast particles in an inhomogeneous medium. Dokl. Akad. Nauk SSSR **134** (1960), 318–321 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1015–1018.

11806:

Kapica, S. P. The radiation from a charge moving in an inhomogeneous medium. Ž. Èksper. Teoret. Fiz. **39** (1960), 1367–1370 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 954–956.

If a charged particle moves through an inhomogeneous medium, the polarization of the medium produced by the charge will lead to radiation of energy by the charge. The author gives a calculation of the effect for a medium with randomly placed inhomogeneities. Formulas are obtained for the intensity, polarization, and directivity of the radiation. It is pointed out that, if the medium contains such inhomogeneities, the emission of Čerenkov radiation will be somewhat altered. *E. L. Hill* (Minneapolis, Minn.)

11807:

Mercier, R. P. Diffraction by finite irregular objects. Proc. Phys. Soc. **77** (1961), 318–327.

Fresnel diffraction theory is employed to derive a theory of statistical correlation between the amplitude of the wave diffracted by an irregular object and the complex amplitude at the screen. The Michelson method for measuring the angular diameter of a story diffraction by a circular object and the reflection of radio waves from the moon are discussed. *W. E. Williams* (Liverpool)

11808:

Fel'd, Ya. N. On a method of solving boundary problems of electrodynamics. Radiotehn. i Élektron. **4** (1959), 2004–2015 (Russian); translated as Radio Engrg. and Electronics. **4**, no. 12, 74–90.

A formal procedure is outlined for finding an expansion for the surface current density on a system of N perfectly conducting bodies of arbitrary shape excited by an incident electromagnetic wave or an impressed e.m.f. The method entails the construction of an orthonormal set of basis vector functions defined on the surfaces, and is therefore not easily applicable in the general case. However, it is applied to the case of N infinitely long parallel strips excited by a plane wave and to N finite hollow coaxial cylinders of arbitrary diameter and arbitrary spacing along the axis excited by an impressed e.m.f. Even in the simplest case of a single element ($N=1$) the solutions for these two problems would require a formidable amount of computation, so that the practical utility of the scheme is somewhat doubtful. *E. T. Kornhauser* (Providence, R.I.)

11809:

Kerker, M.; Matijević, E. Scattering of electromagnetic waves from concentric infinite cylinders. J. Opt. Soc. Amer. **51** (1961), 506–508.

The authors study a generalization of the classic problem of a plane electromagnetic wave scattered by an infinitely long circular cylinder. The cylindrical body is now a set of concentric circular cylinders of different refractive indices. Series expansions involving Bessel and Hankel functions are obtained for the various scattered fields. The usual cases of **E** parallel to the cylinder axes and **H** parallel to the cylinder axis are studied. Only the formed results are outlined and no numerical calculations presented although they would be of great interest.

R. Barakat (Lexington, Mass.)

11810:

Minin, I. N. On the solution of the integral equation for refraction of electromagnetic waves at a boundary. Dokl. Akad. Nauk SSSR **133** (1960), 558–560 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 801–804.

The author discusses some examples of a method for solving integral equations described by V. V. Sobolev [Izv. Akad. Nauk Armyan. SSR. Ser. Fiz.-Mat. Nauk **11** (1958), no. 5, 39–50; MR **21** #276].

A. E. Heins (Ann Arbor, Mich.)

11811:

Jones, D. S.; Noble, B. The low-frequency scattering by a perfectly conducting strip. Proc. Cambridge Philos. Soc. **57** (1961), 364–366.

The scattering coefficient for a plane wave incident upon a narrow, perfectly conducting strip and with small wave number has been given by several authors. The present authors show that the higher-order terms cannot be expanded in the same fashion. Remarks are also made about the same problem for a perfectly conducting elliptic cylinder. *A. E. Heins* (Ann Arbor, Mich.)

11812:

Grinberg, G. A.; Pimenov, Yu. V. The diffraction of electromagnetic waves from infinitely thin perfectly conducting plane screens. Ž. Tehn. Fiz. **27** (1957), 2325–2339.

A new method is proposed for solving problems concerning the diffraction of electromagnetic waves by infinitely thin perfectly conducting screens. The problem is reduced to the solution of two simple independent integral equations. This method eliminates the need to take into account edge effects. It is also shown that if l is a length defining screen dimensions and k the propagation factor, then for sufficiently small values of kl, the solution of the diffraction problem reduces to the solution of successive electrostatic problems. Diffraction by an infinitely thin perfectly conducting disc is worked out to illustrate the method. *J. E. Rosenthal* (Passaic, N.J.)

11813:

Kuricyn, V. N. Arbitrary incidence of a plane electromagnetic wave on a conducting disk. Ž. Tehn. Fiz. **30** (1960), 790–798 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 744–752.

L'auteur applique la méthode générale de G. A. Grinberg et Y. V. Pimenov [#11812]. Le disque est donc supposé parfaitement conducteur et le produit ka inférieur à l'unité ($k=2\pi/\lambda$: constante de propagation de l'onde incidente, a: rayon du disque). La méthode évite l'utilisation de fonctions sphéroïdales, en ramenant la résolution du problème posé à celle de problèmes d'électrostatiques. L'auteur calcule les développements de la densité superficielle de courant sur le disque et de la "surface effective" suivant les puissances successives de ka. Pour la densité de courant, les développements sont arrêtés à $(ka)^4$; pour la "surface effective", qui ne contient que des puissances paires de ka, les développements sont arrêtés à $(ka)^6$.

L. Robin (Paris)

11814:

Ku, Y. H. On nonlinear networks with random inputs. IRE Trans. **CT-7** (1960), 479–490.

11815:

Watanabe, Hitoshi. A computational method for network topology. IRE Trans. **CT-7** (1960), 296–302.

If G is a linear graph with n branches the author associates with any subgraph an n-tuple of zeros and ones — in effect, an n-dimensional vector over the integers modulo 2. Coordinatewise addition, multiplication, and complementation (changing zeros to ones and vice versa) provide the algebraic bases for the computation technique developed by the author. The author introduces the concept of a spread transformation which, essentially, replaces a given tree with a given end-vector by a new tree with one different branch. This is done in terms of the algebraic formalism, and it is shown that all possible trees of a nonseparable graph may be found by a sequence of spread transformations starting from any initial tree. Moreover, with a good choice of the initial tree, "most" trees can be found without duplication.

For networks containing active elements, information depending on an orientation of the graph must also be considered. This is done and the theory is further developed to take account of multitrees and the algebraic signs of the various tree determinants needed in the analysis of active networks.

On the whole, the paper provides an algebraic formalism which is readily adapted to computation and shows that it is adequate to discuss many questions in network topology. *J. B. Giever* (University Park, N.M.)

11816:

Carlin, H. J.; Youla, D. C. Network synthesis with negative resistors. Proc. IRE **49** (1961), 907–920.

This paper treats the properties and synthesis of linear nonreciprocal and active lumped networks. The idealized elements are the positive inductor, positive capacitor, positive resistor, negative resistor, ideal transformer, and gyrator.

The first section of the paper considers the two following closely related subjects: (1) A general method for describing network-terminal properties which is valid even in the absence of the usual immittance and scattering formalisms; and (2) a technique for operating on a network description so that pathologies are removed from such circuits and an immittance or scattering matrix is obtained. The "nullator" and the "norator" are given as examples of pathologic circuits. Three theorems are given in the first section. The first theorem states: The necessary and sufficient condition that: (1) an immittance matrix be constructed from an $n \times 2n[A, B]$ matrix of real, rational functions by adding positive resistors r_k to the n ports of the network; or (2) a rational, real scattering matrix with positive-port normalization numbers r_k is formed from $[A, B]$; $[A, B]$ contains a non-vanishing n-column minor formed of complementary columns from A and B. A more general procedure leads to Theorem 2 for the case where A and B are square matrices and $[A, B]$ is of rank n, and Theorem 3 for the case of any $m \times 2n[A, B]$ matrix of rational fractions with real coefficients.

The second section of the paper considers the synthesis in the frequency domain of a real, rational $n \times n$ immittance matrix (made available by the method of the first section) in which pole locations and pole multiplicities are completely arbitrary. It is shown that such a matrix can be realized with lossless elements and n positive and n negative resistors. Theorems 4 and 5 are given in the

synthesis of networks with lossless elements and negative resistors. Theorems 6 and 7 are given in the discussion on general driving-point immittance functions. Theorems 8 and 9 are given in the synthesis of n ports with n positive and n negative resistors. The paper is very comprehensive and concludes with two appendices dealing with (1) transformation of an $[A, B]$ matrix; and (2) removal of boundary poles from an immittance matrix.

Y. H. Ku (Philadelphia, Pa.)

11817:

Onodera, R. A new approach to Kron's method of analysing large systems. Proc. Inst. Elec. Engrs. C **108** (1961), 122–129.

This paper is essentially a somewhat condensed version of a paper by the author which appeared in the RAAG Memoirs [*RAAG memoirs of the unifying study of basic problems in engineering and physical sciences by means of geometry, Vol. II*, Gakujutsu Bunken Fukyu-Kai, Tokyo, 1958; MR **21** #6758; p. 369]. The main thing of interest is the introduction of a method of analysing electric networks dual to Kron's method of tearing. For this, one should also consult Weinzweig [Quart. Appl. Math. **18** (1960/61), 183–190; MR **22** #3438].

J. B. Giever (University Park, N.M.)

11818:

Livšic, M. S.; Flekser, M. Š. Expansion of a reactive four-terminal network into a chain of simplest four-terminal networks. Dokl. Akad. Nauk SSSR **135** (1960), 542–544 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1150–1152.

It is shown how a reactive four-terminal network can be synthesized by factoring the corresponding transfer matrix ($ABCD$ matrix) and identifying the factors with simple circuits. The factoring is based on a theorem given by Brodskiĭ and Livšic [Uspekhi Mat. Nauk **13** (1958), no. 1 (79), 3–85; MR **20** #7221].

B. Hazeltine (Providence, R.I.)

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

11819:

Корабельщиков, Н. И. [Korabel'ščikov, N. I.]. ★Сборник задач по технической термодинамике [Problem book in engineering thermodynamics]. Izdat. "Rečnoĭ Transport", Moscow, 1961. 126 pp. (1 insert) 0.35 r.

A very conventional, not excessively imaginative collection of standard problems with some solutions in engineering thermodynamics, emphasis being placed on numerical results. In most leading universities in this country the course is now taught at a considerably higher level of understanding, and the collection would be considered antiquated.

J. Kestin (Providence, R.I.)

11820:

Liu, Chen-Ya. On optimum rectangular cooling fins. Quart. Appl. Math. **19** (1961), 72–75.

The problem is as follows: For a fixed fin weight find the fin width for maximum cooling rate. Cooling is by radiation only, and the appropriate heat-conduction equation is written parametrically in terms of the temperature at the end of the fin. Optimisation is obtained by the Lagrange multiplier method.

J. F. Clarke (Cranfield)

11821:

Lewis, J. A. Generalized thermal resistance. Quart. Appl. Math. **19** (1961), 76–80.

Soit V un corps conducteur de surface $S=S_0+S_1+S_2$. La température θ est supposée vérifier les conditions aux limites

$$\begin{aligned} d\theta/dn &= 0 && \text{sur } S_2,\\ &= h_1(\theta_1-\theta) && \text{sur } S_1,\\ &= h_0(\theta_0-\theta) && \text{sur } S_0 \end{aligned}$$

(d/dn dérivée normale; θ_0, θ_1, h_1, h_0, constantes).

Si ϕ est le flux qui entre par S_0 et sort par S_1, on est conduit à généraliser la notion de résistance thermique en posant $R=(\bar\theta_0-\bar\theta_1)/\phi$; $\bar\theta_0$ et $\bar\theta_1$ étant les températures moyennes sur S_0 et S_1.

L'auteur montre que si R_D et R_N sont les valeurs prises respectivement par R pour les conditions $\theta=c^{te}$ et $d\theta/dn=c^{te}$ sur S_0 et S_1 (S_0 et S_1 isothermes dans le premier cas; flux constant sur S_0 et S_1 dans le second cas) on a alors $R_D \leqq R \leqq R_N$.

Ces inégalités s'obtiennent en utilisant la relation de Green qui donne $R=\mathscr{D}/\phi^2$ avec $\mathscr{D}=\int_V (\text{grad}\,\theta)^2\,dv$ et en bornant l'intégrale de Dirichlet $\mathscr{D}$ pour les conditions aux limites en cause.

R. Gerber (Grenoble)

11822:

Bailey, H. R. Condition-convection from a cylindrical source with increasing radius. Quart. Appl. Math. **18** (1960/61), 325–333.

Some particular cases of the system of differential equations

$$\frac{\partial^2 T_s}{\partial r^2}+\frac{1}{r}\frac{\partial T_s}{\partial r}-a^2\frac{\partial T_s}{\partial t}+\frac{\Phi}{k}=\frac{h}{k}(T_s-T)=\frac{P(t)}{r^2}\frac{\partial T}{\partial r}$$

$$(r>0,\quad t>0)$$

in $T_s(r,t)$ and $T(r,t)$ are solved. Here T_s denotes the temperature in a porous solid of infinite extent, and T the temperature in a gas that has penetrated the solid from the axis $r=0$ and burns at the surface of the cylinder $r=r_f$. The source function Φ is proportional to $\delta(r-r_f)$, and r_f is assumed to be either proportional to t or to $\sqrt{t}$. A quasi-stationary state case that reduces to an ordinary differential equation of third order in T, with constant coefficients, is first solved. When $T_s=T$ and r_f is proportional to t, an integral representation of $T(r,t)$ is written and evaluated with the aid of Hankel transforms.

R. V. Churchill (Ann Arbor, Mich.)

11823:

Reiss, H.; Helfand, E. Analytical solutions for some zone melting problems. J. Appl. Phys. **32** (1961), 228–232.

The author gives analytical solutions to the zone melting problem for any distribution of solute for the case of two geometries. One is the infinite straight bar and the other the re-entrant annular ring. A formula is derived which describes the distribution of solute after a number n of molten zones have passed through the system.

R. Gerber (Grenoble)

11824:

Adler, J.; Spalding, D. B. One-dimensional laminar flame propagation with an enthalpy gradient. Proc. Roy. Soc. London Ser. A **261** (1961), 53–78.

Es wird die laminare, eindimensionale, stationäre Flamme untersucht, bei der ausserhalb der Flammenzone ein Enthalpiegradient vorhanden ist. Das Problem führt auf eine nicht-lineare Differentialgleichung zweiten Grades, die mit einer digitalen Rechenanlage numerisch gelöst wird. Bei einem positiven Enthalpiegradienten steigt die Flammengeschwindigkeit an, während bereits bei einem endlichen negativen Enthalpiegradienten die Flammengeschwindigkeit auf Null reduziert wird. Die allgemein gehaltene Randbedingung eines Enthalpiegradienten wird dann auf folgende Beispiele angewandt: (1) Strömung eines brennbaren Gemisches durch einen heissen oder kalten porösen Pfropfen. (2) Flamme zwischen zwei gekühlten Platten (sandwich burner) (3) Verbrennung mit Wärmeabstrahlung (4) Verbrennen eines flüssigen Brennstoffes in einer reagierenden Atmosphäre. Bei allen Beispielen gibt es entweder zwei mögliche Verbrennungsgeschwindigkeiten oder es ist keine stationäre Flamme möglich. *L. Speidel* (Mülheim)

QUANTUM MECHANICS

See also 11567, 11568, 11576, 11782.

11825:

Harig, Gerhard; Schleifstein, Josef (Editors). ★Naturwissenschaft und Philosophie. Beiträge zum internationalen Symposium über Naturwissenschaft und Philosophie anlässlich der 550-Jahr-Feier der Karl-Marx-Universität Leipzig. Akademie-Verlag, Berlin, 1960. 437 pp. DM 9.50.

This volume contains a collection of addresses given on the occasion of the 550th anniversary of the University of Leipzig and organised by the Institute for Philosophy and the Karl-Sudhoff-Institute for History of Medicine and Science. The talks are grouped under three main headings: (1) Philosophical problems of modern physics; (2) Philosophical problems of biology; (3) General philosophical problems of modern science. Those of the first section focus on the problem of causality and its problem in relation to quantum mechanics; a small number deal with relativity theory. The second section includes discussions on the concept of "wholeness", of "life", of reflection action and spontaneity, reversibility and irreversibility. The last section, apart from a lecture on philosophy and cybernetics, is largely devoted to more mundane things such as the place of dialectical materialism in modern science.

It should not be thought that this is the only reference to that concept. On the contrary, its relationship to contemporary science is made a focal point by most of the contributors, the tone being set by the introductory essay of Gerhard Harig. Careful reading of the volume unfortunately does not help to discover how precisely these dialectical concepts are supposed to illuminate the general philosophical problems of science. Where the concept functions seriously at all, it does this more by acting as a kind of direction-giving beacon, aided and abetted by a generous interpretation of dialectical materialism on the point of the individual writer. Thus, to say that "space and time form a mutual dialectic correlation" (p. 35) is to be accommodating indeed. Nor does it "follow" from the assumption that everything is in a constant process of change due to the underlying "matter in motion", that all the resulting objects and phenomena are "hence" in a state of mutual action and reaction and of related interdependence (p. 50). These two examples will show that where the political element intrudes into philosophical argumentation, the result is likely to be flabbiness of reasoning. Nonetheless, where the authors manage to overcome the handicaps of the ensuing tight rope-walking, there often emerge very stimulating ideas.

G. Buchdahl (Cambridge, England)

11826:

Chambers, Ll. G. The geometrical interpretation of Rayleigh's principle and Schwinger's variational principle. Bul. Inst. Politehn. Iaşi (N.S.) **5 (9)** (1959), no. 1–2, 151–160. (Russian and Romanian summaries)

The paper compares and indicates the relationship between the Rayleigh variational principle for the eigenvalues of a vibrating system and the variational principle of Schwinger, for the solution of an integral equation. Geometrical interpretations of both are discussed.

P. M. Morse (Cambridge, Mass.)

11827a:

v. Krzywoblocki, M. Z. On the general form of the special theory of relativity. II. Acta Phys. Austriaca **14** (1961), 22–28.

11827b:

v. Krzywoblocki, M. Z. On the general form of the special theory of relativity. III. On the super-light motion in the general form of the special theory of relativity. Acta Phys. Austriaca **14** (1961), 39–49.

[For part I, see same Acta **13** (1960), 387–394; MR **22** #5439.] The author examines several aspects of the hypothesis that one should consider the set of transformations in which the form $(dx)^2+(dy)^2+(dz)^2-c^2(dt)^2$ is preserved but the value of c may change; c is in any case assumed to be a function of the coordinates x, y, z, t. Actually only transformations involving x and t are investigated, and no effort is made to see whether the set of transformations introduced forms a group nor whether the predictions of new theory agree with known facts about the motions of extreme relativistic particles.

P. G. Bergmann (Syracuse, N.Y.)

11828:

Landhäusser, Erich. Allgemeine Lorentz-Transformation der Dirac-Gleichung. Z. Physik **162** (1961), 438–447.

The Lorentz transformation of the Dirac equation is discussed. The transformation law is given in an explicit form for any finite Lorentz transformation. The result is in the nature of things, and gives no new information.

H. Wakita (Hiroshima)

11829:

Kretschmann, Erich. Grundriss eines Lorentzinvarianten Einbaus der Wellenmechanik in die klassische

Elektronentheorie. Wiss. Z. Martin-Luther-Univ. Halle-Wittenberg. Math.-Nat. Reihe 8 (1958/59), 719–726.

Der verfasser bemüht sich um einen kausalen Einbau der Wellenmechanik in die klassische Elektrodynamik. Dazu wird den Elementarladungen eine Doppelnatur zugeschrieben: sie sind einerseits Sender, andererseits Empfänger von Wirkungen. Als Sender folgen sie den Gesetzen der Elektronentheorie, als Empfänger aber wellenmechanischen Gesetzen. Insbesondere bestünde nach dieser Theorie das H-Atom aus mindestens zwei Punktladungen, die strahlen, jede umgeben von einer Wellenfunktion, welche alle Feldwirkungen empfängt und in Bewegungen der Ladung umsetzt. Träger einer solchen Doppelnatur nennt Verfasser "Letztel". Jedes Letztel wirkt als Sender auf alle anderen Letztel und sich selbst durch das elektromagnetische Feld, das von seiner Punktladung $e \gtrless 0$ und dem mit ihr verbundenen magnetischen Moment ausgesandt wird. Als Empfänger der von allen Letzteln ausgeübten Wirkungen wird das Letztel durch die Anfangsformen $\psi(\mathfrak{R}, 0; \tau)$ einer Schar von Wellenfunktionen $\psi(\mathfrak{R}, t; \tau)$ dargestellt, welche die Letztelweltlinie umgeben (τ = Eigenzeit). Da die gleichzeitige Bestimmbarkeit des Ortes und der Geschwindigkeit des Letztels vorausgesetzt wird, entfällt die übliche quantenmechanische Deutung der für die Wellenfunktionen ψ bestehenden Wellengleichungen. Durch kongruente Abbildung der Ruhräume auf einander gelingt es Verfasser, die Folge der Wellenfunktionen $\psi(\mathfrak{R}, 0; \tau)$, die auf den Ruhräumen ($t = 0$) das Letztel umgeben, zu einer Wellenfunktion $\psi_0(\mathfrak{R}, \tau)$ zu verschmelzen. Diese Verschmelzung wird im Sinne einer fortwährenden "Reduktion" der Wellenfunktion ψ auf immer neue Anfangswerte durchgeführt. Dann werden die Bewegungsgesetze und der Erhaltungssatz von Impuls und Energie des Letztels gewonnen. Die zugehörige Spintheorie wird so entwickelt, daß ein Zerfließen der Wellenfunktion ausgeschaltet wird. Weitere Eigenschaften der Wellenfunktion ψ_0 sollen in einer späteren Arbeit untersucht werden.

M. Pinl (Cologne)

11830:

Costa de Beauregard, O. Harmonie préétablie de la relativité restreinte et des quanta. Sciences (Paris) No. 6 (1960), 31–40.

Author's summary: "Nées l'une l'autre vers 1900 de problèmes d'optique (émission-absorption des ondes d'une part, propagation des ondes de l'autre), la théorie des quanta et la relativité restreinte sont essentiellement deux théories sœurs, comme le proclama la mécanique ondulatoire de Louis de Broglie en 1925. Il fallut pourtant attendre presque jusqu'à 1950 pour voir se constituer la *Théorie quantique des champs* sous forme explicitement relativiste. L'on cherche ici à montrer quels incidents de croissance, les uns techniques, les autres épistémologiques étaient survenus, et quels remèdes ont été apportés."

11831:

Medvedev, B. V. The axiomatic method and the theory of perturbations. Dokl. Akad. Nauk SSSR **135** (1960), 1087–1090 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1206–1209.

In terms of the Bogolyubov quasi-axiomatic approach to quantum fields [cf. N. N. Bogolyubov, B. V. Medvedev, and M. K. Polivanov, *Voprosy teorii dispersionnyh soot-nošenii*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1958; MR **21** #1150], the author establishes a type of formal uniqueness (in perturbation theory) for a quantum field satisfying certain general criteria. No equations of motion or Lagrangian need be specified. More specifically, there can exist in perturbation theory only a finite-parametric family of quantum fields (defined by sets of vacuum expectation values) consistent with given transformation properties for the fields, unitarity of the scattering operator, causality in the relatively strong form given in the cited reference, and for which the vacuum expectation values have at most certain specified polynomial rates of growth. The author notes that the existence of non-trivial quantum fields has not yet been established, and that the question of a permissible choice of growth indices requires further investigation.

I. E. Segal (Cambridge, Mass.)

11832:

Aharonov, Y.; Bohm, D. Time in the quantum theory and the uncertainty relation for time and energy. Phys. Rev. (2) **122** (1961), 1649–1658.

Almost any textbook on quantum theory contains the uncertainty relation $\Delta E \Delta t \gtrsim \hbar$. This relation asserts that a physical system which serves as a clock, defining time to an accuracy Δt, has an uncertainty $\Delta E \gtrsim \hbar/\Delta t$ in its energy. However, it is also often inferred from the above relation that in order to measure energy to an accuracy ΔE, the duration of the experiment must be $\Delta t \gtrsim \hbar/\Delta E$.

The authors of this paper challenge this widespread belief. They remark that since the time of measurement can be represented by an operator belonging only to the observing apparatus, it follows that this time must commute with every operator of the observed system and, in particular, with its Hamiltonian. They conclude that no such limitations on the measurability of energy can be obtained from the mathematical foundations of quantum theory. Finally they devise a Gedankenexperiment showing that energy can be measured reproducibly in an arbitrary short time.

A. Peres (Haifa)

11833:

Stevens, K. W. H.; Josephson, B., Jr. The coupling of a spin system to a cavity mode. Proc. Phys. Soc. **74** (1959), 561–575.

A consistent wave-mechanical formulation of the problem of a spin system interacting with a cavity mode has been presented; the mutual interaction between the spin system and the spin-lattice interaction have been neglected, however. The mathematical complexities, which usually arise in the solution of this type of problem, have been avoided by considering a cavity with perfectly reflecting walls and filled with a dielectric with a finite conductivity. The authors should have examined how far this approximation may deviate from actual reality.

The Schrödinger equation for the damped cavity field together with the interaction terms has been solved, and the complete wave function has been obtained in a closed form. The results indicate that the spin and cavity may exchange energy in a manner analogous to the behaviour of coupled circuits.

S. K. Chakrabarty (Howrah)

11834:

Giambiagi, J. J. Analogy between Lorentz and Foldy-Wouthuysen transformation. Nuovo Cimento (10) **16** (1960), 202.

11835:

Renaud, Paul. Sur l'hypothèse de calculabilité. C. R. Acad. Sci. Paris **251** (1960), 1465–1467.

11836:

Bose, S. K.; Gamba, A.; Sudarshan, E. C. G. Representations of the Dirac equation. Phys. Rev. (2) **113** (1959), 1661–1663.

Authors' summary: "A representation of the Dirac equation which displays its extreme relativistic properties is discussed. The 'spin' appears naturally related to the 'position'."

This paper is closely related to M. Cini and B. Touschek [Nuovo Cimento (10) **7** (1958), 422–423].

S. A. Wouthuysen (Amsterdam)

11837:

Pac, Pong Y. Remarks on the transformation properties of the Dirac equation. Progr. Theoret. Phys. **22** (1959), 857–862.

This is a continuation of an earlier work of the author [Progr. Theoret. Phys. **21** (1959), 640–652; MR **21** #7032] and deals with the properties of dynamical variables in the "extreme relativistic representation", that is, in the representation obtained from that of the usual Dirac equation by the unitary transformation found by M. Cini and B. Touschek [Nuovo Cimento (10) **7** (1958), 422–423] and also discussed by S. K. Bose, A. Gamba and E. C. G. Sudarshan [see #11836]. *N. Rosen* (Haifa)

11838:

Stepanov, K. N. Concerning the motion of a highly relativistic electron in a linear accelerator under the action of an accidental perturbation. Ž. Tehn. Fiz. **30** (1960), 975–980 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 912–917.

Considering the motion of a highly relativistic electron in a linear accelerator of great length, the author examines the possible types of perturbation that may cause noticeable deviations of the particle from the axis of the accelerator. This is assumed to consist of several accelerating gaps, and to work on the travelling wave principle. If there be an accidental error due to the axis of any gap being slightly inclined to the axis of the accelerator, then the electric field of the travelling wave has a transverse component causing a deviation. The other type of perturbation considered is that due to an inexact compensation for the earth's magnetic field, and for iron and steel masses positioned round the accelerator.

In view of the extremely irregular nature of the perturbing quantities, a statistical treatment of the problem, based on Chandrasekhar's stochastic methods, is adopted, and use is made of the Fokker-Planck-Kolmogorov equation to derive the probability density of the magnitudes of physical interest. No results of a numerical nature are indicated, but a special case where the particle energy does not change from a certain point $z=z_1$ on the accelerator to the accelerator end is dealt with. The result derived for this special case is that the deviation of the particle under the action of an "accidental" force grows very rapidly in comparison with the case where the particle energy increases in the section $z>z_1$. As the author himself remarks, this result is however trivial since an increase of energy leads to an increase of mass, but a particle with large mass is deflected less.

B. S. Madhavarao (Poona)

11839:

Scarf, Frederick L. New soluble energy band problem. Phys. Rev. (2) **112** (1958), 1137–1140.

Author's summary: "The Schrödinger equation for $V(x)=-V_0\csc^2(\pi x/a)$ is solved. $E(k)$ and the effective mass are examined along with the wave functions for various values of the parameter (V_0a^2). It is found that the wave functions have intermediate forms even when $m^*\simeq m$, $m^*\simeq\infty$, and that the weak- and tight-binding approximations are not valid in these regions."

A. C. Hurley (Melbourne)

11840:

Zastavenko, L. G.; Čžou, Guan-Čžao. Integral transformations of the I. S. Šapiro type for particles. Ž. Èksper. Teoret. Fiz. **38** (1960), 134–139 (Russian. English summary); translated as Soviet Physics. JETP **11**, 97–100.

This paper is a continuation of a previous paper by the same authors [same Ž. **35** (1959), 1417–1425; MR **21** #5434] in which the wave functions belonging to irreducible representations of the homogeneous Lorentz group are expressed in terms of representations of the homogeneous Lorentz group. The previous paper dealt with particles of non-zero mass, and this paper gives the corresponding treatment for particles of zero rest-mass. A pair of inverse Fourier type integral transforms are given using the same techniques as before, both directly, and by making the transition to the limit $M\to 0$. *C. A. Hurst* (Adelaide)

11841:

Hara, E. Paarerzeugung durch langsame geladene Teilchen im Coulombschen Feld. Z. Naturforschg. **16a** (1961), 155–161. (English summary)

From the author's summary: "The pair creation cross section for heavy particles in the Coulomb field is calculated using Schrödinger-Coulomb wave functions for the heavy incident particles and plane waves for the created pair. It is shown that the order of magnitude of the cross section thus obtained is given by the product of the value deduced by Heitler and Nordheim [J. Phys. Radium **5** (1934), 449–454] with Born approximation, and the Sommerfeld correction factor, which is known for the emission of Bremsstrahlung."

11842:

Jauch, J. M. Systems of observables in quantum mechanics. Helv. Phys. Acta **33** (1960), 711–726.

This paper shows that a quantum-mechanical system which possesses at least one complete commuting set of observables has the property that all its super selection rules commute. In a rough way this is obvious: the operators defining the super selection rules commute with the observables of the assumed maximal commuting set. Thus, they are functions of them and therefore commute with each other. A principal part of the paper is devoted to evolving a mathematically satisfactory formulation of the problem. It is then solved by an application of the theorem: An abelian von Neumann algebra is maximal if and only if it is cyclic. All this is scarcely new but is

important and has been explained nowhere else. In the reviewer's opinion some version of this paper will shortly become standard in quantum mechanics texts with pretensions of precision. *A. S. Wightman* (Princeton, N.J.)

11843:

Datzeff, Assène. Sur l'interprétation de la mécanique quantique. Relations d'incertitude. C. R. Acad. Sci. Paris **251** (1960), 1462–1464.

The author discusses the interpretation of the uncertainty relations in a quantum theory with hidden variables. [See also Bohm, Phys. Rev. (2) **85** (1952), 180–193; MR **13**, 710.] *K. J. Le Couteur* (Canberra)

11844:

Judd, B. R. The matrix elements of tensor operators for the electronic configurations f^n. Proc. Phys. Soc. **74** (1959), 330–339.

To evaluate the splitting of energy levels of rare earth ions in crystals it is useful to know the reduced matrix elements of tensor operators in states of f^n configurations. Since the standard method of evaluation via fractional parentage coefficients is tedious, some simple relations between reduced matrix elements are derived in this paper by means of the following basic group-theoretic theorem. Let A and B be two finite-dimensional vector spaces over the complex field, let $G=\{T\}$ be a group, let $\Gamma_A=\{D^{(A)}(T)\}$ and $\Gamma_B=\{D^{(B)}(T)\}$ be unitary representations of G with representation spaces A and B respectively, let $\alpha=\{a_i\}$ and $\beta=\{b_k\}$ be orthonormal bases of A and B respectively, let $\{H_j\}$ be a finite set of linear operators with domain B and range in A such that

$$D^{(A)}(T)H_jD^{(B)-1}(T) = \sum_{j'} D^{(H)}_{j'j}(T)H_{j'}.$$

where $\Gamma_H=\{\|D^{(H)}_{j'j}(T)\|\}$ is a unitary representation of G, let $M_{ijk}(H)=\langle a_i|H_j|b_k\rangle$, and let $M(H)=\|M_{ijk}(H)\|$ (a column matrix). If A, B, α, β, Γ_A, Γ_B, Γ_H are fixed (but the set $\{H_j\}$ can vary) the number of linearly independent vectors $M(H)$ is not greater than the number of times the trivial 1-dimensional representation ($T\to 1$) of G occurs as a component of $\bar{\Gamma}_A\otimes\Gamma_H\otimes\Gamma_B$. This theorem enables one to relate reduced matrix elements. Since the states of f^n configurations can be characterized in part by the irreducible representations of the 21-dimensional rotation groups R_7 and the 14-dimensional, exceptional, simple, compact group G_2, the preceding theorem is applied to these two groups. Finally, a practical application to the splitting of a dysprosium ion is made.

J. S. Lomont (Yorktown Heights, N.Y.)

11845:

March, N. H.; Murray, A. M. Relation between Dirac and canonical density matrices, with applications to imperfections in metals. Phys. Rev. (2) **120** (1960), 830–836.

By exploiting the relation between the canonical and Dirac density matrices progress is made in the calculation of the Dirac matrix in certain cases. A study is made of density matrices in central field problems. Using the Bloch equation, a new differential equation is derived for the diagonal elements of the canonical density matrix. Free-electron matrices are fully worked out and a perturbation theory based on these free-electron matrices is presented. In the domain of imperfections in metals, a correct formulation to replace Mott's well-known first-order approximation is obtained [N. F. Mott, Proc. Cambridge Philos. Soc. **32** (1936), 281–290].

E. J. Verboven (Utrecht)

11846:

March, N. H.; Murray, A. M. Self-consistent perturbation treatment of impurities and imperfections in metals. Proc. Roy. Soc. London Ser. A **261** (1961), 119–133.

A full perturbation treatment of the Dirac density matrix is developed as a basis for self-consistent calculations in free-electron metals containing localized effects. In this treatment results are used of a paper by the same authors on the relation between Dirac and canonical density matrices [#11845]. The perturbation series is shown to sum to the well-known result for the case of slowly varying potentials. To first order in perturbation theory, exact self-consistent results for the radial density of displaced charge and the Hartree potential in the presence of point singularities are obtained. The density range covers all metals under normal conditions. Mott's first-order method may be approved [N. F. Mott, Proc. Cambridge Philos. Soc. **32** (1936), 281–290]. Finally the plane waves are replaced by Bloch functions for which the present approach is briefly considered. *E. J. Verboven* (Utrecht)

11847:

Keller, Joseph B. Lower bounds and isoperimetric inequalities for eigenvalues of the Schrödinger equation. J. Mathematical Phys. **2** (1961), 262–266.

The author finds explicitly the lowest eigenvalue that a Schrödinger equation $u_{xx}+V(x)u+\lambda u=0$ can have when the value of $\int_{-\infty}^{\infty}|V|^n dx$ (n any real number >1) is prescribed. (The absolute value is omitted. Apparently the author assumes that $V\geq 0$.) This leads to a lower bound for the lowest eigenvalue in terms of this integral. It is shown that these lower bounds are quite accurate in several cases where the eigenvalues are known exactly.

H. F. Weinberger (Minneapolis, Minn.)

11848:

Hamermesh, Morton. Galilean invariance and the Schrödinger equation. Ann. Physics **9** (1960), 518–521.

A simplification and clarification of work of Inönü and Wigner [Nuovo Cimento (9) **9** (1952), 705–718; MR **14**, 352] and Bargmann [Ann. Math. (2) **59** (1954), 1–46; MR **15**, 397] deriving the Schrödinger equation as that defining the only irreducible unitary representation of the Galilean group satisfying certain reasonable physical desiderata. As is customary in the theoretical-physical literature, free algebraic use is made of unbounded operators in Hilbert space, the real-variable questions involved in making a parallel rigorous development being ignored.

I. E. Segal (Cambridge, Mass.)

11849:

Freund, P. G. O. Wave equations invariant under discontinuous groups and the problem of nuclear forces. Acta Phys. Polon. **19** (1960), 139–148.

11850:

Kamefuchi, S.; Takahashi, Y. Invariance and the S-matrix. Nuclear Phys. **17** (1960), 686–694.

It is considered worthwhile to study a quantum-mech-

anical system in which the interaction picture is the basis of the theory without recourse to any other picture. The point is then made that in this picture the Schrödinger equation is formally invariant under arbitrary infinitesimal transformations. This corresponds to the fact that in the conventional formalism the separation of the total Hamiltonian into a free and an interaction Hamiltonian remains completely arbitrary unless a reference is made to the physical implications of this separation. It is then shown that this invariance property leads, under certain conditions, to new kinds of conservation laws whose physical meaning is obscure because the formally conserved quantities are in general non-local which depend on the entire past history of the system.

J. M. Jauch (Geneva)

11851a:

Cornille, Henri. Étude d'une classe de potentiels permettant la résolution par itération de l'équation de Schrödinger. C. R. Acad. Sci. Paris **251** (1960), 2135–2137.

11851b:

Cornille, Henri. Application à une classe de potentiels de la résolution par itération de l'équation de Schrödinger. C. R. Acad. Sci. Paris **251** (1960), 2308–2310.

The method of iteration introduced by Martin [Nuovo Cimento (10) **14** (1959), 403–425; **15** (1960), 99–109; MR **22** #3464, 5382] is extended to the case of non-zero orbital angular momentum by taking Hankel functions as the kernels of the transformations. These transformations are shown to be of the Laplace type, and classes of potentials are found which ensure the existence of these kernels and the convergence of the iterative procedure. In the second note an explicit expression is derived for the kernel in the case of an exponential potential.

A. C. Hurley (Melbourne)

11852:

Bourret, Richard. Autocorrelation of quantum-mechanical wave functions. Canad. J. Phys. **37** (1959), 1062–1067.

Autocorrelation functions are constructed from the state functions of a quantum-mechanical system. Several examples are considered. The examples show that, as regards the amount of physical information which they carry, the autocorrelation functions are intermediate between the state functions and the partition functions.

E. Wolf (Rochester, N.Y.)

11853:

Minnaja, N. Solution of the Schroedinger equation in a constant magnetic field and diamagnetism. Physica **26** (1960), 827–833.

The non-relativistic Schrödinger equation for particles confined to a cylindrical box in a constant magnetic field along the axis, is solved. The eigenvalues are given, and the partition function is written down for a system of subparticles obeying Fermi statistics. In the end, the finite size of the box turns out to be a handicap, and an approximation equivalent to infinite box-size is made. The result, curiously, differs by a factor of 4 from that of Landau for the same case.

H. W. Lewis (Madison, Wis.)

11854:

Isaev, P. S.; Sewerynski, M. Approximate equations for partial π-K scattering amplitudes. Nuclear Phys. **22** (1961), 663–676.

Authors' summary: "Approximate integral equations for partial π-K scattering amplitudes are obtained with the help of the double Mandelstam representation in the low-energy range."

11855:

Deloff, A.; Wrzecionko, J. The phenomenological baryon-baryon scattering theory and the relative parity determination. Nuclear Phys. **20** (1960), 464–474.

Authors' summary: "The reactions $a+b\to c+d$ with four baryons are considered. Two cases of relative intrinsic parities $I_aI_b=\pm I_cI_d$ are taken into account; the phenomenological S-matrix technique is used for obtaining the cross-section and polarizations. An approximation in which, in the initial state, only the S-wave is present is discussed. In this approximation the cross-section for unpolarized particles is isotropic and the polarizations in the final state are zero when $I_aI_b=I_cI_d$ and proportional to $\sin 2\theta$ when $I_aI_b=-I_cI_d$. Some experimental tests for the relative parity determination are proposed."

H. Messel (Sydney)

11856:

Braun, M. A.; Prohorov, L. V. Properties of the scattering amplitude resulting from the unitarity condition. Ž. Èksper. Teoret. Fiz. **39** (1960), 1641–1646 (Russian. English summary); translated as Soviet Physics JETP **12** (1961), 1146–1149.

Authors' summary: "The restrictions imposed on the two-particle scattering amplitude by the unitarity condition are investigated. It is shown that (1) the imaginary part of the scattering amplitude is a bounded function of the energy and possesses no discontinuities of the first kind in the physical region of the variables; (2) as the energy tends to infinity the amplitude cannot grow without bound, except possibly for some particular values of the scattering angle; (3) under certain reasonable assumptions the total scattering cross-section cannot grow indefinitely with increasing energy."

11857:

Minguzzi, A.; Streater, R. F. Dispersion relations for some photon reactions. Nuovo Cimento (10) **18** (1960), 806–812. (Italian summary)

Dispersion relations for fixed momentum transfer are proved here directly from the Jost-Lehmann-Dyson representation, though for a smaller range of momentum transfer than that obtained by continuation in an external mass variable. This is possible only for processes involving photons initially or finally, where the amplitudes are only considered to lowest order in the fine structure constant. The processes considered are photoproduction, Compton scattering, and Bremsstrahlung from nuclei.

John G. Taylor (Baltimore, Md.)

11858:

Minguzzi, A.; Streater, R. F. A note on the proof of dispersion relations. Nuovo Cimento (10) **17** (1960), 946–950. (Italian summary)

The proof of elastic-scattering dispersion relations, for fixed momentum transfer, requires the analyticity of the absorptive part of the scattering amplitude as a function of an external mass. The required region of analyticity is a strip along the real axis, of width independent of the energy. Proofs of the dispersion relations had not shown that the width of the strip was independent of the energy. This independence is shown in this paper by means of the Jost-Lehmann-Dyson representation, for all of the processes considered until now. *John G. Taylor* (Baltimore, Md.)

11859:

Ghosh, P. K. Distribution-theoretic observation on the Born scattering formula (first approximation). Bull. Calcutta Math. Soc. **51** (1959), 199–201.

The first Born approximation in potential scattering is shown to give a physically meaningful result for a large class of singular potentials, if treated by Schwartz's theory of distributions. The exceptions vary either as an ever positive power or as the logarithm of the distance from the scattering centre. *John G. Taylor* (Baltimore, Md.)

11860:

Finn, Albert C. Application of dispersion relations to meson-nucleon scattering. Phys. Rev. (2) **119** (1960), 1786–1802.

This paper contains a careful discussion of the numerical determination of the low-energy meson-nucleon S, P and D phase-shifts from non-forward scattering dispersion relations, assuming charge independence. The number of subtractions necessary is determined by assuming a finite range of interaction, and allows seven of the 10 phase-shifts to be determined. The validity of the partial wave expansions required for separating the different partial waves and evaluating dispersion integrals is discussed in detail; some of these expansions have been justified rigorously, others follow from the Mandelstam representation, and others, involving the high-energy contributions to the dispersion integrals, are justified on the basis of a completely absorbing optical potential used to describe the high-energy scattering. The low-energy dispersion integrals are assumed to be given by the (3, 3) resonance contribution, while corrections from the high-energy integrals are estimated by the optical-potential model, together with scattering in three higher energy resonances.

The numerical predictions from 0 to 280 MeV (pion laboratory kinetic energy) for the unambiguous combinations of phase-shifts are in satisfactory agreement with experimental values, though these latter values are not known with sufficient accuracy to be a critical test for any but the (3, 3) phase-shift.

John G. Taylor (Baltimore, Md.)

11861:

Kibble, T. W. B. Kinematics of general scattering processes and the Mandelstam representation. Phys. Rev. (2) **117** (1960), 1159–1162.

This paper treats a scattering process involving two incoming and two outgoing stable spinless particles. The physical regions of the Mandelstam plane are shown to be bounded by a cubic curve, which is given explicitly. The unitarity conditions are written down in the two-particle approximation, and expressed in terms of integrations over the Mandelstam parameters of the intermediate states. The one-dimensional dispersion relations are used with the unitarity relations, following Mandelstam [Phys. Rev. (2) **115** (1959), 1741–1751; MR **22** #575], and this leads to explicit expressions for the double spectral functions. Most of the derivations and all of the results given in this work exhibit the complete symmetry between the three Mandelstam parameters.

G. R. Allcock (Liverpool)

11862:

Aramaki, Seiya. Dispersion relations and high energy limits in quantum field theory. II. Progr. Theoret. Phys. **25** (1961), 404–410.

This is not only a second part of an article [for part I see Progr. Theoret. Phys. **22** (1959), 485–491; MR **21** #6993] but also an erratum. The result of the first paper is corrected to say that the forward scattering amplitude for pion-nucleon scattering at high energies cannot increase more rapidly than the first power of the energy. This result is based on the assumption that the forward amplitude has no zeros in the complex plane. A lower bound for the asymptotic energy behavior is also given by the result that in the dispersion relations for the sum of the forward scattering amplitudes for positive and negative pions on protons there must be at least one subtraction. The same, however, cannot be said about the difference of the two amplitudes. *M. J. Moravcsik* (Livermore, Calif.)

11863:

Martin, A. S-matrix, left-hand cut discontinuity and potential. Nuovo Cimento (10) **19** (1961), 1257–1265. (Italian summary)

A discussion is given of the analytic properties of the S-matrix for scattering of s-waves in the case of the two-body potential

$$V(r) = \int_{\mu}^{\infty} C(\alpha) \exp[-\alpha r]\, d\alpha.$$

It is pointed out that in the case of this potential, besides the physical branch cut on the positive E axis, there is another, unphysical cut for $-\infty < E < -\mu^2/4$. The problem therefore resembles the one of the s-wave projection of the Mandelstam representation for nucleon-nucleon scattering. It is pointed out that when the discontinuity of the S-matrix across the negative E-axis is known everywhere, a formal relation between $D(\kappa)$, where $2\pi i D(\kappa) = S(i\kappa+\varepsilon) - S(i\kappa-\varepsilon)$, and the scattering amplitude, and between $D(\kappa)$ and $C(\alpha)$, can be attained. These relations are given explicitly, though it cannot be shown in general that the $S(\kappa)$ reconstructed from the $D(\kappa)$ has all the right singular points. When $D(\kappa)$ is not known above $\kappa = \kappa_0$, $C(\alpha)$ for $\alpha < 2\kappa_0$ can still be obtained. *K. Haller* (New York)

11864:

Barut, A. O.; Ruei, K. H. Analytical properties of S matrix and uniqueness of the scattering potential. J. Mathematical Phys. **2** (1961), 181–187.

An account is given of the mathematical properties of the S-matrix for radial wave functions in potential scattering. The authors demand that the asymptotic limit of that solution of the radial Schrödinger equation, which behaves like the Riccatti-Bessel function $j_e(kr)$ near the origin, exists for all complex values of the momentum.

They thereby limit the discussion to potentials for which $\int_0^\infty dr\, r|V(r)|\exp[2\alpha r] < \infty$ for all α; i.e., potentials with Gaussian tails or cut-offs at large r. It is recalled that the Jost functions in this case are entire functions of k^2, the S-matrix is defined everywhere and there are no redundant poles in the S-matrix. A proof is given that under these conditions a unique potential is determined by the phase shifts even when bound states exist. An extension to the relativistic case is discussed. *K. Haller* (New York)

11865:

Moffat, John W. Determination of pion-pion scattering amplitudes satisfying dispersion relations and unitarity. Phys. Rev. (2) **121** (1961), 926–932.

Author's summary: "A method is developed for determining the partial-wave scattering amplitude in terms of the unitarity condition and the known branch cuts and poles of the inverse amplitude. The method is applied to the problem of pion-pion scattering and an implicit solution to the pion-pion partial-wave amplitude is derived for any angular momentum state and for both elastic and inelastic scattering. With the aid of this solution the low-energy resonance behavior of the pion-pion scattering system is studied by neglecting all inelastic processes and concentrating on S- and P-waves. It is found that a P-wave resonance with a position and width required by nucleon electromagnetic structure can be determined in terms of two parameters. An iteration procedure is described that is applicable when the P-wave dominates the equations and this procedure determines the contribution of the unphysical cut. The first iteration of the unphysical cut is numerically integrated on the IBM 709, and the results show that the shift of the resonance position due to the unphysical branch cut can be neglected."

P. Roman (Boston, Mass.)

11866:

Zachariasen, F. Relativistic model field theory with finite self-masses. Phys. Rev. (2) **121** (1961), 1851–1862.

Author's summary: " A model field theory is invented in the following way: Dispersion relations in the energy are assumed to hold for all amplitudes. Unitarity gives the absorptive parts in the 'physical' regions. If it is assumed that the absorptive parts are otherwise zero (in violation of crossing symmetry and the Mandelstam representation), then the dispersion relations and unitarity form an infinite set of coupled integral equations for all amplitudes. An exact solution (at least for the simplest amplitudes) to this set of equations can be found, in which all self-masses, etc., are finite. The solution is equivalent to summing a certain class of Feynman graphs, computed in the usual way. For a wide range of coupling constants, there are no 'ghost' difficulties." *J. M. Jauch* (Geneva)

11867:

Ascoli, R.; Bottino, A.; Molinari, A. Unitarity of the S-matrix and analyticity. I. Nuovo Cimento (10) **19** (1961), 687–695. (Italian summary)

It is shown that, to deduce analyticity properties of the imaginary part of the scattering amplitudes (for two particles) as functions of the momentum transfer, it is only necessary to know the analyticity properties of the production amplitudes as functions of only two dynamical variables, regardless of the number of produced particles.

C. G. Bollini (Buenos Aires)

11868:

Oehme, Reinhard. Continuation of scattering amplitudes and form factors through two-particle branch lines. Phys. Rev. (2) **121** (1961), 1840–1848.

Author's abstract: "It is shown that scattering amplitudes and form factors have two-particle branch lines which connect two Riemann sheets. For partial wave amplitudes and form factors the dispersive parts and, except for square root factors, the absorptive parts are regular functions in the cut energy plane except for isolated poles, physical inelastic cuts and left-hand branch lines. In order to show this it is assumed that, for particles without composite structure, the amplitudes have only such singularities in the physical sheet which correspond to absorptive processes. The analytic properties of absorptive parts are used for a general discussion of structure singularities (anomalous thresholds). It is shown that these structure cuts are extensions of left-hand branch lines in the second Riemann sheet. An example is given of a dispersion relation on the Riemann surface in which the integral over the two-particle branch line is eliminated."

P. Roman (Boston, Mass.)

11869:

Goldberger, M. L. Introduction to the theory and applications of dispersion relations. Relations de dispersion et particules élémentaires (Grenoble, 1960), pp. 15–157. Hermann, Paris; Wiley, New York; 1960.

This chapter contains the course given by the author at Les Houches, in 1960. After a brief history of dispersion relations and a review of classical ideas on the subject, the lectures continue with a detailed account of dispersion relations for the Schrödinger equation. The asymptotic condition, reduction formulae and solution by Green's functions are explained, and dispersion relations are derived by various methods. It is shown that the scattering amplitude satisfies a Mandelstam representation. The double spectral function is determined by the potential and leads to a new method for solving Schrödinger's equation. The presentation of the non-relativistic case is arranged to bring out the analogies with the relativistic case. This part of the work is more or less complete.

The relativistic case is also done in detail, beginning with the reduction formulae. Various dispersion relations are derived, with adequate rigour. It is shown how unitarity leads to integral equations of the Omnès type, which are solved. Dispersion relations are derived for forward scattering and are applied to the π-n system. The rest of the article contains a discussion of spin and iso-spin analysis, kinematics and crossing relations for the π-n system, ending with an account of projection into partial waves, and briefly, non-forward relations. Applications of the Mandelstam representation are dealt with by G. F. Chew in another chapter.

The article is written with stress on the physical meaning at all times; the style is humorous and informal. Topics are dealt with thoroughly and there is some repetition (for example, reduction formulae are derived several times "just for practice"). It is a pity such an important set of lectures was not printed in a more elegant type.

R. F. Streater (Upton, N.Y.)

11870:

Schumacher, Clifford R.; Bethe, Hans A. Usefulness of polarized targets and the polarization transfer tensor in

reconstruction of the nucleon-nucleon scattering matrix. Phys. Rev. (2) **121** (1961), 1534–1541.

From the authors' summary: "A method is presented for removing the ambiguities arising in the reconstruction of the nucleon-nucleon scattering matrix from data. . . . The method would provide an independent means of arriving at the correct set of phase-shift solutions, and is based on a knowledge of the polarization transfer tensor which may be determined from triple scattering experiments or from double scattering measurements on nucleons scattered on a polarized target. The use of polarized targets would have many experimental advantages. They would permit determination of (1) the depolarization tensor D for large scattering angles, without requiring the measurement of the polarization of a very slow nucleon, (2) the correlation tensor by a cross-section measurement instead of the difficult simultaneous measurements of the polarization of both the final nucleons, and (3) the 'difficult' components of A' and R' of D by the measurement of the polarization of an initially unpolarized beam scattered by a polarized target."

R. F. Streater (Upton, N.Y.)

11871:

Fujii, Yasunori. Isobar model and the S-wave pion-nucleon scattering. Progr. Theoret. Phys. **24** (1960), 1013–1032.

Author's abstract: "For the scattering lengths of the S-wave pion-nucleon scattering, Chew, Goldberger, Low and Nambu [Phys. Rev. (2) **106** (1957), 1337–1344; MR **19**, 920] have shown that the result of their dispersion relation without subtraction exhibits an excellent agreement with the observations, as long as the integrations are carried out over the 3-3 resonant state alone. Correspondence of their result to the Hamiltonian formalism is examined by replacing the 3-3 resonant state by an isobaric particle of spin 3/2, which is described by the Rarita-Schwinger theory. The scattering amplitudes are calculated by means of the lowest-order perturbation theory. By requiring further that the amplitudes should decrease to zero in the high-energy limit, it is found necessary to introduce interaction terms other than the conventional one. These terms are left undetermined from the location and the width of the 3-3 resonance. By the appropriate choice of the parameters involved in these terms, CGLN's result can be reproduced in the narrow width approximations."

E. C. G. Sudarshan (Rochester, N.Y.)

11872:

Sunakawa, Sigenobu. On the theory of rearrangement collisions. Progr. Theoret. Phys. **24** (1960), 963–979.

The scattering matrix elements corresponding to a rearrangement of particles during an atomic collision are discussed. These processes can be dealt with in a manner similar to the ordinary theory of potential scattering. The unitarity of the S-matrix gives the generalized optical theorem.

S. A. Wouthuysen (Amsterdam)

11873:

Tani, Smio. Structure of the S-matrix in the presence of a bound state. Phys. Rev. (2) **121** (1961), 346–349.

This paper is a continuation of earlier work by the author [Phys. Rev. (2) **117** (1960), 252–260; MR **22** #543] where the orthogonality phase shift, arising from the possibility of bound-state formation, was introduced. The author discusses, in the paper under review, the factorisation of the S-matrix into an orthogonal part containing the effects of the bound state, and a residual part. If a Born expansion can be applied to the residual part, agreement with Watson's result [ibid. **88** (1952), 1163–1171] on final state interactions is obtained. An experimental test for the factorisation is given by the threshold electrodisintegration of the deuteron into the triplet S-state, which is not allowed if the final state interaction is determined by the total phase shift, but is allowed if the author's factorisation is correct.

John G. Taylor (Baltimore, Md.)

11874:

Okubo, Susumu; Feldman, David. Some aspects of the covariant two-body problem. I. The bound-state problem. Phys. Rev. (2) **117** (1960), 279–291.

The authors investigate the Bethe-Salpeter equation but with the interaction generalized to include not only the conventional ladder approximation but also a similar pair annihilation diagram. They first show that the new diagram gives no effect except for S-states. For all states with higher angular momentum the problem considered here reduces to the problem discussed by Wick [Phys. Rev. (2) **96** (1954), 1124–1134; MR **16**, 655] and Cutkosky [ibid., 1135–1141; MR **16**, 656] a few years ago. For S-states the authors write the Fourier transform of the "wave function" $\psi(p)$ in the following form:

$$\psi(p) = \iint_0^\infty dx\,dy \times \frac{2H(x,y)}{[x(p^2+\frac{1}{4}E^2+m^2+pE)+y(p^2+\frac{1}{4}E^2+m^2-pE)+1]^3}.$$

The weight function $H(x, y)$ is considered as an unknown quantity to be determined from the Bethe-Salpeter equation. The vector E is the total energy momentum in the center of mass system and m is the nucleon mass. To obtain this equation the authors have made certain simplifying assumptions, viz., that all particles involved are considered as scalar and that the mass of the meson is put equal to zero. The end of the paper contains a discussion of what happens when these assumptions are relaxed and gives a clear impression of how necessary it is to make them in order to get manageable equations. This "Ansatz" is a generalization of the technique used by Wick and Cutkosky and also has a certain similarity to other spectral representations which are known to appear in perturbation-theory computations. The Bethe-Salpeter equation implies that the weight H must fulfil a certain differential equation. The authors find

$$\frac{\partial^2 H(u,v)}{\partial u\,\partial v} = \frac{g^2}{4\pi^2}\,\frac{H(u,v)}{(\frac{1}{4}E^2+m^2)(u+v)^2-\frac{1}{4}E^2(u-v)^2+u+v}$$

with the boundary conditions $H(u=0, v)=H(u, v=0)=1$. Here, g is the coupling constant. This differential equation for H can be used to determine the eigenvalue $\varepsilon=\sqrt{-E^2}$. The authors find that the binding energy is given by the elementary Bohr formula plus correction terms which are computed in terms of an expansion in the parameter g^2/m^2. Terms of the fourth and fifth order in this parameter are given explicitly in the paper.

G. Källén (Lund)

11875:

Okubu, Susumu; Feldman, David. Some aspects of the

covariant two-body problem. II. The scattering problem. Phys. Rev. (2) **117** (1960), 292–306.

Continuing their earlier work [cf. #11874] the authors consider the scattering of two scalar nucleons interacting through a zero-mass scalar meson field. In contrast to the previous paper the authors here restrict themselves to the conventional ladder approximation. They find that the "Ansatz" they used before does not work for the scattering problem and has to be generalized to

$$F(p,q,E)=\iiint_0^\infty dx\,dy\,dz\,z^2H(x,y,z)\exp\{-iz\times[(p^2-k^2+pE)x+(p^2-k^2-pE)y+(p-q)^2+\mu^2]\}.$$

Here, F is a function which is essentially the Fourier transform of the two-nucleon Green's function. The letter k indicates the relative momentum in the center of mass system while E is the total energy momentum vector in the same coordinate system. The mass μ is the meson mass which finally is put equal to zero. The symbol q is the variable of the Fourier transformation. For $q=k$, F reduces essentially to the Fourier transform of the Bethe-Salpeter "wave function". By formal operations the authors find that the scattering cross-section can be related to a certain limiting form of the function H through the equation

$$\frac{d\sigma}{d\Omega}=4\pi^6(-E^{-2})\left|\int_0^\infty dz H(u=\infty,v=\infty,z)\times\exp\{-iz[(k-k')^2+\mu^2]\}\right|^2.$$

The initial and final relative momenta are here denoted by k and k' respectively. In trying to apply this formula to the problem under discussion, the authors find that the limiting form of H indicated here does not exist when the meson mass is put equal to zero. This corresponds to the well-known infrared divergence which one always finds in scating problems of this kind [cf., e.g., J. Schwinger, Phys. Rev. (2) **75** (1949), 898–899]. To get some insight in the handling of this problem the authors discuss non-relativistic Coulomb scattering using a similar formalism as for the field-theoretical problem. They find that there also one encounters a similar difficulty and that it can be related to the well-known distortion of the plane wave with a logarithmic phase due to the long range of the Coulomb interaction. For the non-relativistic problem they find that they can get rid of this difficulty with the aid of a suitable renormalization technique for the Schrödinger wave function. For the field-theoretical problem which is the main subject of the paper they find that a similar renormalization technique only helps in the first approximation but that the infrared term remains in the radiative corrections. The authors state that this term "should be canceled by taking into account the cross-section for the emission of soft photons". *G. Källén* (Lund)

11876:

Streater, R. F. On the unphysical region in dispersion relations. Nuovo Cimento (10) **13** (1959), 57–76. (Italian summary)

In order for a dispersion relation to have physical content, a way must be found to express the integrand in the unphysical region in terms of physical data. For π-N scattering, H. Lehmann [Nuovo Cimento (10) **10** (1958), 579–589] has shown how to do this. For K-N scattering, the problem is harder because of contributions from inelastic processes like $K+N\to\pi+\Lambda$ (the small contributions from $\Lambda+2\pi$ states are not included) at complex values of the momentum transfer and sub-threshold values of the energy.

This paper shows how, in principle, the problem is to be solved. A set of coupled dispersion equations and unitarity relations for the other amplitudes involved (like $K+N\to\pi+\Lambda$) is derived. In these equations, the "known" quantities are to be determined by Lehmann's method, and the solution of these equations gives the analytic continuation required for the unphysical region of the original dispersion relation. The procedure is justified rigorously, but only for those fictitious values of the K and Λ masses for which, at present, the K-N scattering dispersion relation can itself be proved.

J. C. Taylor (Rochester, N.Y.)

11877:

Ekstein, H. Equivalent Hamiltonians in scattering theory. Phys. Rev. (2) **117** (1960), 1590–1595.

It is shown that in a non-relativistic many-particle theory in second quantized form, in which no bound-state solutions occur and in which the interaction Hamiltonian annihilates all zero and one-particle states, there is a redundant set of Hamiltonians corresponding to the same S-matrix. The members of this set are related by $H^1=UHU^{-1}$, where U is unitary. The conditions on U are very general and do not require it to obey any dynamical principles, so that it is clear that there are many different U's. However, a Hamiltonian with a simple two-body local potential is, after such transformation, no longer of that form. This result is therefore consistent with the theorem that, in the absence of bound states and in the case of a local potential, the S-matrix determines the Hamiltonian.

The redundancy is essentially due to the strong convergence of the exact wave-functions to the asymptotic states, which are products of one-particle wave functions. It is shown that the difference between the original and the transformed S-matrices disappears because of the applicability of the Riemann-Lebesgue lemma to the strongly convergent expressions that arise in this formulation.

It is also shown that this redundancy reflects an ambiguity in the formal description of the particle creation operators, so that particles described by UaU^{-1} are scattered precisely like those described by a.

K. Haller (New York)

11878:

Layzer, Arthur J. Free propagator expansion in the evaluation of the Lamb shift. I, II. J. Mathematical Phys. **2** (1961), 292–307, 308–323.

In the first paper detailed mathematical methods are given for the purpose of evaluating the terms of order $\alpha(\alpha z)^6\ln^2(\alpha z)$ and $\alpha(\alpha z)^6\ln(\alpha z)$ in the Lamb shift calculations. In the second paper these results are applied for the calculation of the Lamb shift in the ground-state of H, D, and He^+ with the following results in megacycles per second (exper. = experimental, theor. = theoretical).

	H	D	He^+
Exper.	1057, 77 ± 0, 10	1059, 00 ± 0, 10	14040, 2 ± 4, 5
Theor.	1057, 70 ± 0, 15	1059, 08 ± 0, 16	14047, 2 ± 3, 0

J. M. Jauch (Geneva)

11879:

Neuman, M. Causality and the Lorentz group. Nuovo Cimento (10) **19** (1961), 443–466. (Italian summary)

The author redefines causality and relates his new definition to the Lorentz group. He introduces a transition amplitude S for his actual physical system and also a transition amplitude for free fields s. If now S is expanded with respect to s, with expansion coefficients τ, the latter are found to depend, inter alia, on a generalized radiation kernel D. The requirement of Lorentz covariance and causality now applies to each many-fields kernel ${}^{m}D^{n}$, so that each admits the Lorentz group and vanishes for space-like separations of initial and final states.

P. G. Bergmann (Syracuse, N.Y.)

11880:

Chew, Geoffrey F.; Mandelstam, Stanley. Theory of the low-energy pion-pion interaction. Phys. Rev. (2) **119** (1960), 467–477.

This is the first concrete realization of the Heisenberg S-matrix program within a manifestly covariant framework. Unitarity alone (or even with relativistic invariance) does not couple states of different angular momenta, for example, in a general theory of two-particle scattering; this information would be provided if one could consider the two-particle scattering amplitude as an analytic function of energy and momentum transfer and determine this analytic function in terms of all its singularities. Such a program is carried out in this paper for the case of pion-pion scattering with a framework which extends relativistic invariance into a manifestly covariant substitution law. This places "a powerful new condition on the combined energy and angular momentum dependence of the amplitude; such a condition . . . is not known outside field theory." Specifically, the Mandelstam prescription for extending the scattering amplitude to complex values of the energy and momentum [S. Mandelstam, Phys. Rev. (2) **112** (1958), 1344–1360; **115** (1959), 1741–1751; MR **20** #5057; **22** #575] is used to write down partial wave dispersion relations and to obtain the imaginary parts of the real-analytic function representing the partial wave amplitude on the "left-hand cut". An integral equation for the partial wave amplitude is used to transcribe the information as obtained. One has the discontinuities on the "left-hand cut" as an integral over the imaginary part for positive values of the energy summed (with appropriate weights) over various angular momentum states; the summation over the various partial waves is suitably approximated in the actual calculation. The discontinuities on the right-hand cut are known from unitarity.

A general method of solving such integral equations (the "N/D method") is developed. The pion-pion coupling constant is introduced by normalizing the scattering amplitude at a suitable point. "The order of magnitude of the new constant is established; and a procedure for solving the integral equations by iteration is outlined." Some discussion of the nature of the solution is also presented. [Compare a somewhat related formulation by Cini and Fubini, #11882.] This paper initiates a project in which great emphasis is placed on dynamical systematization of a limited but important class of amplitudes in terms of general qualitative assumptions and a minimum number of parameters. *E. C. G. Sudarshan* (Rochester, N.Y.)

11881:

Chew, Geoffrey F.; Mandelstam, Stanley; Noyes, H. Pierre. S-wave dominant solutions of the pion-pion integral equations. Phys. Rev. (2) **119** (1960), 478–481.

This paper presents numerical solutions of the coupled integral equations for pion-pion scattering derived in the previous paper [#11880]. Changes of variable are made to facilitate the numerical integration and then results are obtained by a straightforward iterative process. Results are given for the range of λ, the pion-pion coupling constant, which give consistent solutions of the type required: this range is from -0.46 to $+0.3$. Only the simplest type of solution is obtained, that dominated by the S-wave amplitude. Solutions of other types are discussed briefly.

D. F. Mayers (Oxford)

11882:

Cini, M.; Fubini, S. Theory of low-energy scattering in field theory. Ann. Physics **10** (1960), 352–389.

An approximation scheme expected to be valid for "low-energy" scattering is developed starting from the Mandelstam representation and assuming that the "nearest singularities" dominate the behaviour of the low-energy amplitudes. With these approximations one obtains coupled integral equations (in terms of a real weight function) which are equivalent to the Chew-Mandelstam equations [#11880] and can be transformed into their final equations. Here, however, there is the great advantage that one does not have to consider explicitly the unitarity condition in the unphysical region, and the derivations and approximations are more transparent. Various special cases are briefly discussed and a more realistic and comprehensive study is promised.

E. C. G. Sudarshan (Rochester, N.Y.)

11883:

Efremov, A. V.; Meshcheryakov, V. A.; Shirkov, D. V.; Tzu, H. Y. On deriving equations from the Mandelstam representation. Nuclear Phys. **22** (1961), 202–206.

This paper criticises the proposal of Chew and Mandelstam [#11880] for solving their integral equations for various scattering processes. It is argued that the partial wave expansion will be a poor approximation in the unphysical region. This is illustrated by expanding a simple function with the correct analytic properties. So the authors bring to attention their own proposal [Efremov, Meščeryakov and Širkov, Ž. Èksper. Teoret. Fiz. **39** (1960), 438–449; MR **22** #9129] to use dispersion relations only at those angles for which the method works. They remark that this gives enough information since at low energies only a few angular momentum states count.

R. F. Streater (Princeton, N.J.)

11884:

Simonov, Yu. A.; Ter-Martirosyan, K. A. Semi-analytical solution of equations of the Chew-Mandelstam type. Ž. Èksper. Teoret. Fiz. **39** (1960), 1442–1449 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 1003–1007.

A simple method is developed, whereby one may rapidly obtain adequate approximations to the solutions of the S-wave dominant pion-pion equations of Chew and Mandelstam [#11880]; Chew, Mandelstam and Noyes [#11881]. The numerator of the integrand appearing in the contribution of the left-hand cut of each reciprocal amplitude is approximated by a polynomial in a suitable

variable. This permits the integrals over the left-hand cut to be evaluated, giving analytic expressions for the amplitudes. Self-consistency conditions are then imposed. Convergence is rapid. The results obtained by keeping only the leading term of each polynomial are plotted alongside the curves obtained numerically by Chew, Mandelstam and Noyes, and lie very close to the latter.

G. R. Allcock (Liverpool)

11885:

Wakita, Hitoshi. Mathematical foundation of the quantum theory of free fields. Progr. Theoret. Phys. **24** (1960), 555–568.

This is a treatment of free fields which keeps as close as possible to the usual non-rigorous treatment, but is mathematically well-defined.

The author considers a set of field operators $q_\alpha(x)$ defined at every point of space-time, and satisfying the commutation rules $[q_\alpha(x), q_\beta^*(x)]_\pm = \delta_{\alpha\beta}$. From the set of Hilbert spaces $\mathfrak{h}(x)$ generated by each set $q_\alpha(x)$, $q_\alpha^*(x)$, an incomplete direct product $\mathfrak{h}(\sigma) = \otimes_{x\in\sigma}\mathfrak{h}(x)$ is formed. σ is a space-like surface. $\mathfrak{h}(\sigma)$ is well known to be nonseparable, but in this paper the author defines a linear space $\mathfrak{H}_\mathfrak{S}(\sigma)$ whose completion is separable. $\mathfrak{H}_\mathfrak{S}(\sigma)$ is spanned by the set of antilinear functionals $\Psi(\sigma)$, defined over $\mathfrak{h}(\sigma)$, for which all the components $\Psi(\sigma; 0) = (\Omega_0(\sigma), \Psi(\sigma))$, $\Psi(\sigma; \alpha, x) = (q_\alpha^*(x)\Omega_0(\sigma), \Psi(\sigma))$, etc., belong to the class $\mathfrak{S}$ of indefinitely differentiable and rapidly decreasing functions. ($\Omega_0(\sigma)$ is the vacuum state in $\mathfrak{h}(\sigma)$.) If $\theta(x)$ is a positive definite distribution, an inner product in $\mathfrak{H}_\mathfrak{S}(\sigma)$ is defined by

$$(\Psi_2(\sigma), \Psi_1(\sigma)) = \bar{\Psi}_2(\sigma; 0)\Psi_1(\sigma; 0) + \sum_\alpha \delta_x (\bar{\Psi}_2(\sigma; \alpha, x) * \Psi_1(\sigma; \alpha, x)) + \cdots$$

and for the usual cases of scalars, spinors, etc., the completion $\mathfrak{H}(\sigma)$ of this space is separable.

The usual commutation rules containing singular distributions appear as the symbolic representation of products of operators defined on different spaces $\mathfrak{H}_\mathfrak{S}(\sigma)$ and $\mathfrak{H}_\mathfrak{S}^*(\sigma)$. The latter space is dual to $\mathfrak{H}_\mathfrak{S}(\sigma)$ and is spanned by the set of functionals whose components belong to the set $\mathfrak{D}_c'$ of rapidly decreasing distributions.

One interesting suggestion is the possibility of an unambiguous definition of a state $\Psi(\sigma; \alpha, x; \beta, x)$ with a norm $\sum_{\alpha,\beta}\delta_x(\bar{\Psi} * \Psi)$, for such a state corresponds to the product $q_\alpha^*(x)q_\beta^*(x)$, and this is usually considered to be meaningless.

C. A. Hurst (Edinburgh)

11886:

DeWitt, Bryce S. Quantization of fields with infinite-dimensional invariance groups. J. Mathematical Phys. **2** (1961), 151–162.

Author's summary: "A general approach to the problems of quantizing fields which have infinite-dimensional invariance groups is given. Space and time are treated on a completely equal footing. A Poisson bracket is defined by means of Green's functions, independently of the discovery or recognition of canonical variables, and is shown to satisfy all the usual identities. In accordance with the measurement theoretical foundations of the quantum theory, the Poisson bracket (i.e., commutator) is defined only for physically measurable group invariants. The Green's functions give a direct description of the propagation of small disturbances arising from a pair of mutually interfering measurements.

"In order to establish a correspondence between this approach and conventional canonical theory, a motivation for the adopted definition of the Poisson bracket is outlined with the aid of the fundamental theorem of canonical transformation theory. The rest of the discussion is logically independent of this, however. The general theory of 'wave operators' and their associated Green's functions is briefly reviewed. Specific details connected with the group theoretical side of the theory are handled in such a way that problems of constraints are completely avoided. In the last section the general method is applied to the Yang-Mills field, as a nontrivial example. The problem of factor ordering is not studied."

S. Deser (Waltham, Mass.)

11887:

Ruckenstein, Éli; Westfried, Florin. Le coefficient de transfert de masse ou de chaleur fluide-particule pour les systèmes multi-particulaires à distribution homogène. C. R. Acad. Sci. Paris **251** (1960), 2467–2469.

En utilisant un modèle hydrodynamique simple les auteurs établissent une expression du coefficient de transfert fluide-particule pour un système multi-particulaire à distribution homogène, pour $\mathrm{Re} < 1$. Cette formule se trouve être en bonne concordance avec les données expérimentales de B. J. Gaffney et T. B. Drew [Indust. Engrg. Chem. **42** (1950), 1120–1127].

R. Gerber (Grenoble)

11888:

Salzman, George; Salzman, Freda. Solutions of the static theory integral equations for pion-nucleon scattering in the one-meson approximation. Phys. Rev. (2) **108** (1957), 1619–1628.

The one-meson approximation to the Low equations for elastic pion-nucleon scattering consists of three coupled singular linear integral equations for three scattering functions. These equations cannot be expected to be accurate for high energy, but the present paper investigates their usefulness for low-energy scattering (up to 170 Mev). The authors have made a detailed study of the numerical solution of these equations for various values of the coupling constant and of the cut-off constant which is introduced to ensure convergence of the integrals. A particular choice of these constants gives good agreement with experiment for scattering in the (3, 3) state, is consistent with data for the (3, 1) state, but predicts phase shifts for the (1, 1) state which seem too large. The paper concludes with a quantitative discussion of the insufficiency of the static model and of the effect of nucleonic recoil.

A. J. Coleman (Kingston, Ont.)

11889:

Jahn, Helmut. Über die Durchführung einer Näherung der starken Kopplung beim Chew-Low-Formalismus. Z. Physik **156** (1959), 633–656. (English summary)

G. and F. Salzman's results [#11888] show that the Chew-Low treatment of π-n scattering does not predict the observed shape of the lowest resonance curve. According to the author's summary in the present paper: "A strong-coupling approximation method for the Chew-Low

meson-nucleon scattering formalism is developed, considering, for simplicity, the case of charged scalar mesons. As a starting point, the Low equation formulated for the case of strong coupling is stated. This formulation explicitly takes into account the formation of excited compound nucleon states which arise in the case of strong coupling.... The one-meson approximation is constructed; this corresponds to a certain kind of many-meson approximation of the usual Low equation whose limit as $g \to \infty$ holds exactly. It is hoped that these considerations will serve to guide an analogous investigation using the pseudoscalar theory." *A. J. Coleman* (Kingston, Ont.)

11890:

Dürr, H.-P. Isospin und Parität in der nichtlinearen Spinortheorie. Z. Naturforschg. **16a** (1961), 327–345. (English summary)

Continuing earlier work [cf., e.g., H.-P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, same Z. **14a** (1959), 441–485; MR **21** #6244] the author discusses certain symmetry properties of the fundamental equation

$$\gamma \frac{\partial \psi(x)}{\partial x} \pm l^2 \gamma_5 \gamma_\mu \psi(x)[\bar{\psi}(x)\gamma_5\gamma_\mu\psi(x)] = 0.$$

It is first argued that the best way to interpret the operator $\psi(x)$ is to understand it to be a two-component Weyl spinor under ordinary Lorentz transformations and to have similar transformation properties also under rotations in isospin space. It is then suggested that parity should be defined with the aid of the transformation $\bar{x} \to -\bar{x}$; $x_0 \to x_0$; $1 \to -1$, i.e., the length l appearing in the "coupling constant" is also given a change of sign. The author then introduces a Tamm-Dancoff approximation scheme consistent with the symmetry assumptions and computes mass values of some elementary particles in the model. The reliability of the various approximations and other assumptions is very difficult to judge.

G. Källén (Lund)

11891:

Zumino, B. The renormalization constants in quantum electrodynamics. Nuovo Cimento (10) **17** (1960), 547–557. (Italian summary)

The author extends the discussion of the consistency with gauge invariance of the theorem of Källén, namely that

$$\operatorname*{Lim}_{(p-q)^2 \to \infty} \Gamma_\mu^{\mathrm{Ren}}(p, q) \to Z_1 \cdot \gamma_\mu$$

where p^2 and q^2 are on the mass shell. The difficulty is that, while the left side of the equation is invariant if the photon lines undergo the transformation

$$D_{\mu\nu} \to D_{\mu\nu} + k_\mu k_\nu M,$$

the constant Z_1 on the right side alters. Hence the theorem can hold only with respect to a single choice of M. While this does not mean that the theorem is necessarily invalid, it makes one rather uneasy, since the theorem was derived by using the same gauge (choice of M) as that in which Z_1 is defined as the vertex renormalization constant.

The conjecture is made that an inconsistency could possibly be found by showing with methods similar to those of Lehmann, Symanzik and Zimmermann [Nuovo Cimento (10) **2** (1955), 425–432; MR **17**, 927] that

$$\operatorname*{Lim}_{p^2 \to \infty} \Gamma_\mu^{\mathrm{Ren}}(p, q) \to 0.$$

In this limit (off the mass shell) it can be shown with Ward's identity that

$$\operatorname*{Lim}_{p^2 \to \infty} \Gamma_\mu^{\mathrm{Ren}}(p, q) \to Z_1 \cdot \gamma_\mu.$$

This result is consistent with the transformation properties of the photon lines given above. If the limits are compared, one would find an inconsistency with the assumption of a non-vanishing value for Z_1.

K. Johnson (Cambridge, Mass.)

11892:

Evans, L. E.; Fulton, T. Asymptotic conditions in quantum electrodynamics. Nuclear Phys. **21** (1960), 492–507.

The asymptotic condition is proposed to hold in quantum electrodynamics in the so-called "Coulomb" (radiation) gauge formulation of the theory. The formal aspects of renormalization are studied. No discussion is made of the main difficulty of such a condition in electrodynamics, namely, that the long range of electrostatic forces (or in other language, the infra-red divergence) complicates the asymptotic limiting procedures (charged particles are never, even asymptotically, "free"). The renormalization factor ($N(p)$) for the electron field is defined by the equation

$$\langle 0|\psi'|p\rangle = N(p)\langle 0|\psi^{(\mathrm{in})}|p\rangle$$

where ψ' is the unrenormalized field, $|p\rangle$ is the state of a single particle with momentum p, and $\langle 0|\psi^{(\mathrm{in})}|p\rangle$ has the form of the matrix element for a free field. The implication is made that $N(p)$ is diagonal with respect to the spin matrices, whereas a second-order calculation indicates otherwise.

K. Johnson (Cambridge, Mass.)

11893:

Evans, Lawrence E. High energy theorem in quantum electrodynamics. Nuclear Phys. **17** (1960), 163–168.

The theorem proved is that the conventional vertex function (that defined by Dyson) must vanish in the limit of infinite-momentum transfer also in quantum electrodynamics. The theorem was proved for meson theory by Lehmann, Symanzik and Zimmermann. The method used is that of LSZ. The proof differs only in minor respects from that of LSZ essentially because the equation of continuity enables one to represent tensor functions (e.g., $\langle 0|j^\mu j^\nu|0\rangle$) in terms of a single invariant function rather than two. This single function has the same properties of positive definiteness as that for a spinless field.

K. Johnson (Cambridge, Mass.)

11894:

Zimmermann, W. One-particle singularities of Green's functions in quantum field theory. II. Nuovo Cimento (10) **16** (1960), 690–704. (Italian summary)

[For part I, see Nuovo Cimento (10) **13** (1959), 503–521; MR **21** #6996.] Time-ordered Green's functions are expanded in such a way as to exhibit their one-particle singularities. The argument is based on general principles but the answer is the one to be expected on the basis of perturbation theory.

J. C. Polkinghorne (Cambridge, England)

11895:

Sekine, Katsuhiko. Canonical and non-canonical method of quantisation in the Lee model. Nuclear Phys. **23** (1961), 245–268.

This paper describes and elaborates an idea suggested some time ago by Heisenberg and collaborators [Z. Naturforschg. **14a** (1959), 441–485; MR **21** #6244]. In the non-linear spinor model of elementary particles investigated by these authors one encounters the problem that the (anti-) commutator between two field operators vanishes for equal times. Further, it fulfils a certain differential equation with the equal-times value of the commutator as initial condition. The question is: How is it possible to get a non-vanishing solution of the differential equation with such an initial-value problem? As the "realistic" theory investigated by the Heisenberg school is much too complicated to allow an explicit solution, the author of the present paper investigates instead the Lee model with suitably adjusted parameters. There, he is able to work with an exact solution and does not have to rely on approximations which are difficult to justify. The main idea is to say that the concept of an equal-times commutator or, more generally, even the concept of a field at a given time is something very singular and also somewhat unphysical. Instead, one should average all the fields over a small but finite time interval Δt and work only with these quantities. E.g., the Hamiltonian should be constructed from the averaged fields and so on. The author shows by explicit construction that it is possible to replace the usual canonical formalism by a "non-canonical quantization" of this kind. The physical consequences of the two formalisms are the same—at least for energies E fulfilling $E \ll 1/\Delta t$. Further, with such a formalism it is not surprising that one can also have a non-vanishing value of the commutator for unequal times when it vanishes for equal times. The initial-value problem in the new formalism involves the field operators in the whole time interval Δt and not only at a given time.

G. Källén (Lund)

11896:

Montaldi, E. On the structure of the four point functions in Heisenberg's non linear spinor theory. Nuclear Phys. **23** (1961), 439–451.

The author sets himself the problem of writing down a representation for the vacuum expectation value of the product of four spinor fields in the Heisenberg theory of elementary particles. (A summary of this theory is given, e.g., in W. Heisenberg, Rev. Mod. Phys. **29** (1957), 269–278 [MR **19**, 813] and W. Heisenberg and collaborators, Z. Naturforschg. **14a** (1959), 441–485; [MR **21** #6244].) The representation is supposed to fulfil the usual conditions of general field theories and also the particular symmetry properties of the Heisenberg model. After some formal manipulations the author gets a result which involves only two spectral functions integrated over certain mass parameters. For some reason or other the author is not interested in finding a representation of the most general function fulfilling his assumptions but is satisfied with a particular solution. As an example of this we can mention that local commutativity is handled by a technique due to V. N. Gribov [Ž. Èksper. Teoret. Fiz. **34** (1958), 1310–1318; MR **22** #3471] and the result suffers from the same shortcoming as does Gribov's result. The analytic function obtained in x-space is regular in the product of the cut planes for all the complex variables involved. At least for the three-point function it is known that this result does not follow from the general assumptions one would like to use [G. Källén and A. Wightman, Mat.-Fys. Skr. Danske Vid. Selsk. **1** (1958), no. 6, 1–58; MR **22** #3470]. It is most improbable that the corresponding large domain is generally correct for the four-point function investigated in this paper. As a matter of fact, this is also admitted by the author in a footnote. However, from the general style of the main text one gets a strong impression that this footnote has been added after the main body of the paper was written.

G. Källén (Lund)

11897:

Ruelle, D. Connection between Wightman functions and Green functions in p-space. Nuovo Cimento (10) **19** (1961), 356–376. (French and Italian summaries)

This is a systematic study of the Wightman functions $W(z)$ (the analytic continuation, in the complex coordinate space, of the vacuum expectation values of products of field operators), the Green function $G(k)$ and the connection between them. The properties of a single $W(z)$, derivable from the axioms of quantum field theory, are stated. It is shown that the vacuum expectation value of the time-ordered product of field operators can be continued, in the complex momentum space, to an analytic function, which is the author's definition of the Green function. All its boundary values at real points, which include the retarded functions introduced by H. Lehmann, K. Symanzik and W. Zimmermann [Nuovo Cimento (10) **6** (1957), 319–333; MR **19**, 1133] and many others in general, are expressed in terms of the boundary values of $W(z)$. Here the concept of the cycles in combinatorial topology is used as a convenient tool. Several properties of $G(k)$ are derived and the equivalence between these properties of $G(k)$ and the aforementioned properties of $W(z)$ is established. For the most part, the author considers $W(z)$ or $G(k)$ as an analytic function of the zeroth component of z or k with other components remaining real. This necessitates a non-trivial proof of the Lorentz invariance of $G(k)$ but is an essential trick in the author's approach to the problem.

The same subject has been studied also by O. Steinmann [Helv. Phys. Acta **33** (1960), 257–298, 347–362; MR **22** #2373, 3474], H. Araki [#11898], and N. Burgoyne [unpublished as yet].

H. Araki (Kyoto)

11898:

Araki, Huzihiro. Generalized retarded functions and analytic function in momentum space in quantum field theory. J. Mathematical Phys. **2** (1961), 163–177.

From the author's introduction: "The retarded functions in quantum field theory are, as is well known, boundary values of an analytic function in momentum space. In this paper, we will attempt a systematic investigation of this analytic function and its boundary values. Such an investigation has also been made independently by Ruelle, Steinmann, and Burgoyne [D. Ruelle, Thesis, Brussels, 1959; O. Steinmann, Helv. Phys. Acta **33** (1960), 347–362; MR **22** #3474; N. Burgoyne (private communication)]. The present work puts emphasis on the geometrical nature of the problem. First we consider the analytic function in the energy component only, and we easily obtain all its boundary values, which include all the conventional retarded and advanced functions. We obtain

necessary and sufficient conditions for generalized retarded functions to be obtainable from Wightman functions satisfying the usually considered conditions, namely, (W1) relativistic covariance, (W2) local commutativity or anti-commutativity, and (W3) certain mass-spectrum conditions. The resulting conditions on the r function are (R1) relativistic covariance, (R2) support properties in x-space (retardedness or advancedness), (R3) two-term identities in momentum-space for momentum below threshold, (R4) four-term identities [O. Steinmann, Helv. Phys. Acta **33** (1960), 257–298; MR **22** #2373]. The properties (R1)–(R3) are translatable into a statement about the domain of analyticity of this analytic function. Namely, it is analytic in the union of various extended tubes plus points of contact of two neighboring tubes for real parts of one momentum below threshold." *S. Azuma* (Fukusima)

11899:

Kawarabayashi, Ken; Machida, Shigeru. Electromagnetic structure of the nucleon. Progr. Theoret. Phys. **25** (1961), 17–34.

The authors deduce spectral representations of the electromagnetic form factors of the nucleon for fixed momentum transfer q^2 on the basis of general principles. That twelve functions G_i are considered from Lorentz covariance, of which one variable is chosen as $-p'^2$ (p': final nucleon momentum), and that unphysical mass restriction is unnecessary for the proof owing to $q^2>0$, are main differences from the treatment adopted by G. F. Chew et al. [#11900]. It is shown that in the lowest mass approximation the absorptive part is expressed in terms of the $I=J=\frac{1}{2}$ state only; therefore the contributions from resonance states are expected in higher mass configurations. *N. Kumasawa* (Tokyo)

11900:

Chew, Geoffrey F.; Karplus, Robert; Gasiorowicz, Stephen; Zachariasen, Fredrik. Electromagnetic structure of the nucleon in local-field theory. Phys. Rev. (2) **110** (1958), 265–276.

Making use of the dispersion-relation method, the authors first derive the mass spectral representations for the nucleon-form factors. After discussing possible intermediate states which contribute to the distribution functions, they carry out a specific evaluation for two-pion contribution, expanding the integrands in inverse powers of the nucleon mass. If the expressions are cut off, the results of the static model can be reproduced. For this evaluation a critique based on the perturbation theory has been made by K. Hiida et al. [Progr. Theoret. Phys. **22** (1959), 247–273; MR **21** #6987].

N. Kumasawa (Tokyo)

11901:

Taketani, Mituo; Katayama, Yasuhisa. On a unified description of elementary particles. Progr. Theoret. Phys. **24** (1960), 661–678.

In this paper an attempt is made to describe the elementary-particle interactions by means of the (Λ, n, p) and (μ, e, ν) triplets, using the Sakata model for strong interactions and the symmetry between the two triplets [Gamba, Marshak, and Okubo, Proc. Nat. Acad. Sci. U.S.A. **45** (1959), 881–885; MR **21** #3242] for the weak interactions. The interactions are written down, and then a brief discussion is given of how the mass differences between the members of each triplet may arise.

John G. Taylor (Baltimore, Md.)

11902:

Federbush, P. Minimal electromagnetic coupling for spin two particles. Nuovo Cimento (10) **19** (1961), 572–573. (Italian summary)

The author considers the free-spin two-field with non-zero mass (which has $2s+1=5$ degrees of freedom); upon coupling it minimally (i.e., via $\partial_\mu \to \partial_\mu - ieA_\mu$) to an external electromagnetic field, the number of degrees of freedom seems to increase to 6. To restore the correct number of variables (and so remove this inconsistency), a term linear in $F_{\mu\nu}$ and bilinear in the spin-two amplitude must be added to the Lagrangian. Further, as in other higher-spin cases, the "minimal" coupling prescription is not unique; it depends on the choice of free-field Lagrangians which differ among themselves by pure divergences. *S. Deser* (Waltham, Mass.)

11903:

Braun, M. A.; Novožilov, Yu. V. Some properties of dressed particle operators in field theory. Ž. Èksper. Teoret. Fiz. **39** (1960), 1317–1322 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 919–922.

The authors study dressed particle operators, defined as producing physical one-particle states from the physical vacuum. After giving some equivalent definitions of these operators, and showing that states made by these operators at a single time t converge strongly for $t\to\pm\infty$ to the corresponding out- or in-states, the authors show, using perturbation theory, that the dressed operators cannot commute for space-like separation of their arguments unless they are the free field. {Reviewer's comment: This last conclusion also follows, without using perturbation theory, from the work of P. G. Federbush and K. A. Johnson, Phys. Rev. (2) **120** (1960), 1926 [MR **22** #7731].}

O. W. Greenberg (Cambridge, Mass.)

11904:

Cook, J. M. Asymptotic properties of a boson field with given source. J. Mathematical Phys. **2** (1961), 33–45.

A quantized scalar field with a given external source interacting linearly with the field is a formally solvable problem. The total Hamiltonian H_{tot} is reduced to the free Hamiltonian $H_{\text{free}}+\text{const}$ by a canonical transformation of the field variables obtained from a shift of the origin to remove the linear term from the quadratic form H_{tot}. The present paper is devoted to the mathematical justification of these formal manipulations.

A quantized boson field is represented explicitly by giving the creation and annihilation operators in Fock-space γ consisting of the infinite direct sum of symmetrized tensor products of a Hilbert-space $\mathfrak{H}$ with itself. To every element $\phi\in\mathfrak{H}$ there corresponds a creation and annihilation operator $C^*(\phi)$ and $C(\phi)$. The group of unitary transformations U on $\mathfrak{H}$ is represented by a group $\Gamma(U)$ of unitary transformations on γ. In particular the group which is generated by the single particle Hamiltonian H is represented by a group, the generator of which is the free Hamiltonian $H_{\text{free}}\equiv d\Gamma(H)$ of the quantized field.

A total Hamiltonian H_{tot} is considered which differs

from the free Hamiltonian by a linear term in the creation and annihilation operators. The following results are then established:

(1) The two Hamiltonians H_{free} and H_{tot} are unitarily equivalent.

(2) The operator $\exp\{iH_{\text{tot}}\,t\}\exp\{-iH_{\text{free}}\,t\}$ converges for $t\to\pm\infty$ weakly to the operator $\exp[-(\phi,\ \phi)/4]\times\exp[ip(\phi)]$ where $p(\phi)=i[C(\phi)-C^*(\phi)]^{\sim}/\sqrt{2}$ and $\sim$ is the mapping which assigns to a closable operator its closure.

(3) The transformation operator

$$U(t_2,t_1)\equiv\exp\{iH_{\text{free}}\,t_2\}\exp\{iH_{\text{tot}}(t_1-t_2)\}\exp\{-iH_{\text{free}}\,t_1\}$$

satisfies weak $\lim_{t\to\infty}U(t,\ -t)=\exp[-(\phi,\ \phi)/2]I$. The considerations are then extended to a time-dependent course. For the particular case of a source which is switched on and off "adiabatically" by choosing $\phi=\phi_\tau=\exp(-\alpha|\tau|)$, $\alpha>0$, one obtains a transformation operator $U(t_2,\ t_1)$ which satisfies strong $\lim_{t\to\infty}U(t,\ -t)=I$. Thus in this case the weak limit can be replaced by the strong limit and the numerical factor $\exp[-(\phi,\ \phi)/2]$ (corresponding to a "wave function renormalization" in the language of formalistic field theory) is replaced by 1. In both cases the S-matrix exists and it is equal to the unit matrix in agreement with the well-known result which is obtained by purely formal manipulations.

J. M. Jauch (Geneva)

11905:

Terleckiĭ, Ya. P. Statistical theory of the nonlinear field. Dokl. Akad. Nauk SSSR **133** (1960), 568–571 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 812–815.

The probability density $W\{\psi(x)\}$ for a set of c-number fields, collectively denoted by $\psi(x)$, is taken proportional to the canonical distribution $\exp[bS\{\psi(x)\}]$ when the system is in equilibrium with a generalized thermal bath for which b can be complex, and where S denotes the action functional. If S describes a nonlinear field theory, important fields are bounded and represent particles in that these fields are concentrated near isolated world-lines. Hence $S\{\psi(x)\}\to S\{x(t)\}$, where $x(t)$ represents the parametric description of the spatial coordinates of the particles. Adopting the Feynman path integral formalism [R. P. Feynman, Rev. Mod. Phys. **20** (1948), 367–387; MR **10**, 224] the author remarks that "all the results of quantum theory are obtained . . . if we assume that this (nonlinear classical) field is in statistical equilibrium with a thermal bath having constant imaginary temperature". Such a bath would arise from "a surrounding universe filled with particles of imaginary mass", while the constant temperature follows because "particles of imaginary mass cannot carry entropy".

J. R. Klauder (Murray Hill, N.J.)

11906:

Umezawa, H.; Kamefuchi, S. Equivalence theorems and renormalization problem in vector field theory (The Yang-Mills field with non-vanishing masses). Nuclear Phys. **23** (1961), 399–429.

The authors study the equivalence theorems and the renormalizability problem mainly in the case of the Yang-Mills field with non-vanishing masses. They conclude that the theory is not renormalizable, and that the origin of the non-renormalizability can be traced back to the mass terms.

H. Watika (Hiroshima)

11907:

Nishijima, K. Asymptotic conditions and perturbation theory. Phys. Rev. (2) **119** (1960), 485–498.

Author's summary: "The formulation of field theories based on few postulates but not using Lagrangian, Hamiltonian, or field equations has been investigated by many authors. In such a theory the coupled integral equations for Green's functions play the role of a substitute for field equations and serve to determine various physical quantities from the theory. In this paper we give a general prescription for the systematic solution of the coupled integral equations in perturbation theory.

"We discuss two kinds of Green's functions: (1) the retarded functions, and (2) the time-ordered functions. Especially in the solution of the latter we have to use dispersion relations, and one finds a complete correspondence between the numbers of subtractions in the dispersion relations and the types of interactions in the conventional field theory; furthermore, the so-called renormalized coupling constants can be introduced into our theory through the boundary conditions supplementing the subtracted dispersion relations. On the contrary, however, it is not possible even to define the unrenormalized coupling constants in our scheme. In other words, all unobservable divergent quantities are completely eliminated in our formulation, and no divergences occur in the course of the entire calculations.

"We apply this method to quantum electrodynamics to illustrate the above statement and also to show how one can get convergent unambiguous solutions in agreement with the conventional renormalized quantum electrodynamics. In discussing quantum electrodynamics, it is necessary to discover how to express the requirement of gauge invariance without referring to Lagrangian, Hamiltonian, or field equations; it is found that a set of equations which is a straightforward generalization of the Ward identity meets this requirement."

A. Salam (London)

11908:

Nishijima, K. Time-ordered Green's functions and electromagnetic interactions. Phys. Rev. (2) **122** (1961), 298–306.

A new formulation of relativistic quantum field theory proposed by Nishijima [#11907] and Muraskin and Nishijima [#11909] is applied to a gauge invariant system. In order to take into account the gauge invariance property, the W-T equations discussed in #11907 are utilized. It is shown that the conventional renormalized quantum electro-dynamics can be reproduced by (1) unitarity, (2) a parametric dispersion relation and (3) a set of the W-T equations, up to the lowest few orders of the perturbation expansion. The problem of the "elementarity" of a particle and related problems are also discussed.

Y. Takahashi (Dublin)

11909:

Muraskin, M.; Nishijima, K. Time-ordered Green's functions and perturbation theory. Phys. Rev. (2) **122** (1961), 331–340.

The second author has proposed an elegant formulation of relativistic quantum field theory based only on a few plausible postulates, without recourse to Lagrangian, Hamiltonian, or field equations [#11907]. This is further elaborated in the present paper. The basic idea is roughly,

in the terminology of the conventional field theory, as follows.

If we know the S-matrix up to a certain order in the coupling constant, the unitarity condition would allow us to determine the absorptive part of the S-matrix in the next order. In order to determine the dispersive part of the S-matrix in this order, however, we need another condition, i.e., a dispersion relation. As a relation sufficient to serve this purpose, a "parametric dispersion relation" is taken. A new relativistic quantum field theory is thus formulated by starting with the fundamental postulates (1) unitarity, (2) parametric dispersion relation.

The equivalence to the conventional renormalized field theory is illustrated by perturbation expansion.

It should be noticed, however, that the above postulates are essentially classical, and the quantum condition or its equivalent condition must be smuggled in somehow, in addition to them. Furthermore, the position of the well-known relation between spin and statistics, which is one of the remarkable consequences of the usual relativistic quantum field theory, should also be clarified in this formalism. These two points are unfortunately not discussed. *Y. Takahashi* (Dublin)

11910:

Arnous, E.; Heitler, W.; Takahashi, Y. On a convergent non-local field theory. I. Nuovo Cimento (10) **16** (1960), 671–682. (Italian summary)

This paper starts out with the point of view that the divergences in quantum field theory must be avoided even at the expense of Lorentz invariance. A suitable modification of the usual theory is to have a non-local Hamiltonian in the interaction picture of the form

$$H(x) = \int \bar{\psi}(x')F(x-x', x-x'', x-x)\psi(x'')\phi(x)d^3x'd^3x''d^3x,$$

where F is some form factor. Except in the local limit where $F(x, x', x'')=\delta(x)\delta(x')\delta(x'')$, the theory is not Lorentz-invariant. In view of the physical success of the relativistic renormalization theory, the reason for introducing such a theory is merely a mathematical one. The theory contains no way of calculating F, and so is mainly phenomenological.

It is found that a conserved charge can be defined, just as in the local theory. The Schrödinger equation can always be formulated in an invariant manner for any group of transformations, but this does not imply the invariance of the S-matrix. The requirement that S be invariant up to second order in F leads to conditions on F, which are not sufficient to ensure that local commutativity holds up to second order.

R. F. Streater (Princeton, N.J.)

11911:

Arnous, E.; Heitler, W.; O'Raifeartaigh, L. On a convergent non-local field theory. II. Nuovo Cimento (10) **16** (1960), 785–805. (Italian summary)

The general theory of I [see #11910] is applied to electrodynamics. The Lorentz condition and gauge-invariance condition are formulated as conditions on the form-factor. It is found that $F_{\mu\nu}$, j_μ are exactly covariant quantities only in the local limit. Worse, $F_{\mu\nu}$ is not exactly gauge invariant, so new gauge-invariant field strengths are defined. From this is defined a current which differs from the usual one even in the local limit.

The treatment of the energy-momentum vector P^μ is satisfactory. This stems from the fact that $F(x-x', x-x'', x-x)$ is invariant under translations. There is no discussion of angular momentum operators. Finally, an example of a form-factor F is given which ensures that every order of the perturbation series is finite (in this the present theory differs from previous attempts at a non-local theory). The resulting S-matrix is not gauge invariant or Lorentz invariant above first order. No discussion is given of the convergence of the perturbation series, and it is not shown that high-order terms are small.

R. F. Streater (Princeton, N.J.)

11912:

Tzou, K. H. Quelques propriétés de symétrie intrinsèques vis-à-vis du groupe d'inversion en théories des particules et le rapport spin-statistique. J. Phys. Radium **22** (1961), 142–148. (English summary)

Author's summary: "In the general theories of particles with field equations of a certain type we study at first some intrinsic symmetry properties of certain matrix operators in connection with each of the inversion operations. With the aid of these intrinsic properties, it is shown that the antiunitary-unitary inversions or the unitary-antiunitary ones can all determine the statistics of elementary particles according to their spin, if we postulate the invariance of the field quantization rules under any one of these operations. The unitary-unitary inversions of the antiunitary-antiunitary ones, on the contrary, are incapable for the determination of particle statistics. It is particularly remarked that the $CPTM$ invariance, like the CPT invariance, is not an automatical invariance in quantum theory of fields; they are assured in this case only when elementary particles obey their respective good statistics according to their spin."

11913:

Dalgarno, A.; McNamee, J. M. The Hartree perturbation method for helium. Proc. Phys. Soc. **77** (1961), 673–676.

The Hartree-Fock procedure has been extended to determine the effects of perturbations on atomic systems. The accuracy of such results depends upon the accuracy of the wave functions for the unperturbed system. Recently a twelve-parameter representation of the unperturbed orbital for helium has become available. In this paper, this orbital is used to compute the dipole and quadrupole polarizabilities of helium as well as the nuclear shielding factor. The results are compared with other computed values and with experimental results. The method yields lower bounds for the polarizabilities; the absolute error in any of the predicted values is less than 4.3%. *C. Froese* (Vancouver, B.C.)

11914:

Cohen, M.; Dalgarno, A. Stationary properties of the Hartree-Fock approximation. Proc. Phys. Soc. **77** (1961), 748–750.

The theorem is proved that the first-order correction to the expectation value of any operator which can be expressed as a sum of one-electron operators, vanishes if

Hartree-Fock wave functions are used in its evaluation. Expectation values for helium of some one-electron operators computed from Hartree wave functions are compared with those computed from accurate wave functions of Pekeris. *C. Froese* (Vancouver, B.C.)

11915:

Burns, Gerald; Wikner, E. G. Antishielding and contracted wave functions. Phys. Rev. (2) **121** (1961), 155–158.

Contracted wave functions, in which some correction is made for the effect of the external field acting on an ion, are used to calculate atomic polarisabilities and anti-shielding factors for a number of ions.

D. F. Mayers (Oxford)

11916:

Dalgarno, A.; Stewart, A. L. A perturbation-variation calculation of eigenvalues. Proc. Phys. Soc. **77** (1961), 467–470.

Authors' summary: "A variational method, based upon perturbation theory, is used to calculate the eigenvalue of the helium sequence. With a trial wave function containing essentially only one variable parameter, the resulting errors range from 0.3 ev for H^- to 0.01 ev for Ne^{8+}."

D. F. Mayers (Oxford)

11917:

Huzinaga, S. Analytical methods in Hartree-Fock self-consistent field theory. Phys. Rev. (2) **122** (1961), 131–138.

Two methods are described for applying a variational process to the energy of a many-electron system with orbitals expanded in terms of basic functions. The methods are applicable to systems including two open shells, in cases where orthogonality problems have to be considered.

D. F. Mayers (Oxford)

11918:

Bose, Asok K. Über den Einfluss der Zwei-Teilchen-Spin-Bahn-Wechselwirkung auf die Winkelverteilung und die Polarisation am Deuteron elastisch gestreuter Neutronen. Z. Naturforschg. **16a** (1961), 95–112.

Bekanntweise ist die Annahme von Zentralkräften auch schon in der Theorie des Deuterons nicht ganz ausreichend. So muss man z. B. zur Erklärung des beobachteten Quadrupolmomentes auch schwache Nichtzentralkräfte einführen, die man meistens in der Form von Einteilchen-Spin-Bahnwechselwirkungen ansetzt. Selbstverständlich müssen in Atomkernen jedoch auch Zweiteilchen-Spin-Bahnwechselwirkungen (z. B. in der Theorie der Energieniveaus, der magnetischen Momente usw.) auftreten. Ziel der vorliegenden Arbeit ist solche Kräfte in die Theorie der elastischen Streuung von Neutronen an Deuteronen einzuführen. In der Einleitung bespricht der Verfasser die bis jetzt vorliegenden experimentellen Resultate und kommt zu dem Schluss, dass es keine ganz gute Übereinstimmung mit der heutigen Theorie gibt.

Den Ausgangspunkt der Rechnungen bildet die Wellengleichung des auftretenden Dreikörperproblems

$$(1)\quad \{-\hbar^2(2M)^{-1}(\Delta_1+\Delta_2+\Delta_3)+V_{12}+V_{13}+V_{23}\}\cdot\psi = (E_{Lab}+\varepsilon)\psi,$$

wo E_{Lab} die Energie des einfallenden Neutrons und ε die Bindungsenergie des Deuterons bedeutet. Die Wechselwirkungen V_{ij} werden in zwei Teile zerlegt

$$(2)\quad V_{ij} = V_{ij}^{S}+\lambda V_{ij}^{SB},$$

in der das erste Glied die Zentralkräfte enthält, die weiter aus Wignerschen, Majoranaschen, Bartlettschen und Heisenbergschen Kräften bestehend angenommen werden. V_{ij}^{SB} bedeutet dagegen die Zweiteilchen-Spin-Bahnwechselwirkungen. Durch Einführung von neuen Koordinaten wird die Schwerpunktsbewegung abgespalten und durch weitere Vereinfachungen erhält man dann eine Gleichung, welche im ersten Teil der Arbeit mit Hilfe der Bornschen Näherung verhältnismässig einfach gelöst wird. Als interessantes Ergebnis folgt, dass erstens keine Interferenz zwischen den Beiträgen des Zentralpotentials und der Spin-Bahnwechselwirkung auftritt und zweitens dass die letztere Wechselwirkung die Vorwärtstreuung nicht beeinflusst. Eine Polarisation der Neutronen tritt ebenfalls nicht auf. Im zweiten Teil wird mit Hilfe der "No-Polarisation" Approximation gerechnet, d.h. es wird angenommen, dass das einfallende Neutron den Zustand des Deuterons nicht beeinflusst. Diese Berechnungen sind viel weitläufiger. Nach entsprechender Umformung der Ausgangsgleichung werden die Matrizenelemente für Zentralkräfte und für Spin-Bahnkräfte hergeleitet. Aus den Berechnungen folgt, dass die Bahnanteile der direkten Spin-Bahnkräfte durch das über die Wellenfunktion des Deuterons gemittelte Potential $(l(l+1))^{1/2}V_2(r)$ wirken und dass die Wirkung der Bahnanteile der Austausch-Spin-Bahnkräfte wesentlich verschieden von der der direkten Spin-Bahnkräfte und der der Austausch-Zentralkräfte ist. Selbstverständlich stellt die Spin-Bahnkopplung nur eine kleine Korrektur dar. *T. Neugebauer* (Budapest)

11919:

Serdobolsky, V. I. Dispersion formulas in the theory of nuclear reactions. Nuclear Phys. **21** (1960), 245–255.

The outlines of a unified theory of nuclear reactions are presented. A selection criterion is developed which singles out the "compound nucleus resonances" and enables the display of the Green's function in terms of a set of poles corresponding to these levels and their complex reduced half-widths. The derivations do not make use of the concept of the radius of the auxiliary nuclear hypersurface; hence it is stated that these formulae "take into account the tails of wave functions responsible for multi-particle processes occurring close to the surface of the nucleus".

E. C. G. Sudarshan (Rochester, N.Y.)

11920:

Gupta, Suraj N. Relativistic one-pion exchange nuclear potential. Nuclear Phys. **24** (1961), 160–162.

Author's summary: "The exact relativistic one-pion exchange nuclear potential is derived in the centre-of-mass system, and the relationship between the relativistic potential and the Schrödinger equation is discussed."

P. Chevallier (Strasbourg)

11921:

Beliaev, S. T. Concerning the calculation of the nuclear moment of inertia. Nuclear Phys. **24** (1961), 322–325.

11922:

Goto, Junko; Machida, Shigeru. Nuclear forces in the

momentum space. Progr. Theoret. Phys. **25** (1961), 64–82.

This work presents the preliminary step for treating the two-body system in momentum-space, without expanding the potentials with respect to the inverse of the nucleon mass. This work is just the same as the work by P. S. Signell [Progr. Theoret. Phys. **22** (1959), 492–498], except that in this work the one-pion exchange potential is explicitly expressed in the form given by L. Wolfenstein and J. Ashkin [Phys. Rev. (2) **85** (1952), 947–949], and L. Eisenbud and E. P. Wigner [Proc. Nat. Acad. Sci. U.S.A. **27** (1941), 281–289].

T. Sasakawa (Cambridge, Mass.)

11923:

Hall, G. G. The stability of a wavefunction under a perturbation. Philos. Mag. (8) **6** (1961), 249–258.

Author's summary: "The concept of the stability of a wavefunction under a perturbation is defined and the properties which distinguish stable wavefunctions are deduced. The stability of approximate wavefunctions is particularly examined and a number of general results stated. Self-consistent wavefunctions are shown to be stable under one-electron perturbations. The concept enables many hitherto isolated results to be unified and extended. In particular a new method of calculating nuclear magnetic shielding constants for atoms is given. The concept also suggests means of improving an approximate wavefunction so that the derived estimate of some property is made more accurate."

A. C. Hurley (Melbourne)

11924:

ter Haar, D. On the density matrices used in Hartree-Fock calculations. Physica **26** (1960), 1041–1044.

It is pointed out that the density matrices derived from Hartree-Fock wave-functions differ from the density matrices introduced in statistical mechanics. A nomenclature which distinguishes these two concepts is suggested.

A. C. Hurley (Melbourne)

11925:

Yeddanapalli, L. M. Quantum theory of valency. J. Annamalai Univ. Part B Sci. **22** (1960), 17–46 (1961).

This series of four lectures provides an elementary introduction to the quantum theory of valency. After a discussion of simple harmonic motion, the Schrödinger equation is introduced and applied to the hydrogen atom and molecule. This leads up to a discussion of spin valency, the valence-bond theory and molecular orbital theory. The final lecture deals with hybridization of atomic orbitals and applications to aromatic compounds.

A. C. Hurley (Melbourne)

11926a:

Bopp, Fritz. Lorentzinvariante Wellengleichungen für Mehrbahnsysteme. Bayer. Akad. Wiss. Math.-Nat. Kl. S.-B. **1958**, 167–220.

11926b:

Laugwitz, Detlef. Anhang: Über Deltafunktionen. Bayer. Akad. Wiss. Math.-Nat. Kl. S.-B. **1958**, 220–225.

An attempt to formulate a theory of elementary particles based on the Heisenberg [Rev. Mod. Phys. **29** (1957), 269–278; MR **19**, 813] hypothesis that the neutrino is the elementary "glue" from which all other fundamental particles can be obtained by fusion. A Fock-space treatment, based on Dirac many-time amplitudes, is the basis for the relativistic description of these neutrino quanta. It is assumed that these amplitudes, $\psi_n(x) = \psi_{n,\nu_1\nu_2\cdots\nu_n}(x_1, x_2, \cdots, x_n)$, satisfy the following system of equations:

$$(1)\qquad \sum_{i=1}^{m} (\gamma_i^\mu \partial_\mu)\psi_m(x) = \sum_n \int d^{4n}x' V_{mn}(x, x')\psi_n(x')$$

where $V_{mn}(x, x')$ is the Lorentz-invariant interaction operator. It is then argued that in the absence of real processes a system consisting of one neutrino is described by the amplitudes $\psi_1(x)$ satisfying $\gamma^\mu \partial_\mu \psi_1(x) = 0$; a system consisting of two neutrinos, by an amplitude $\psi_2(x_1, x_2)$ satisfying the equation

$$(2)\qquad (\gamma_1^\mu \partial_{1\mu} + \gamma_2^\mu \partial_{2\mu})\psi_2(x_1, x_2) = V(x_1, x_2)\psi_2(x_1, x_2),$$

etc. This equation for the two neutrino system is analysed in detail for the case $V(x_1, x_2) = \varepsilon^4 W \delta(x_1 - x_2)$, where W is a constant matrix (i.e., $W = a_1\gamma_{1\mu}\gamma_2^\mu + a_2\gamma_{15}\gamma_2^5 + \cdots$). The conservation laws are derived for this situation and various solutions for ψ_2 investigated. (Due to the singular nature of the interaction, ε is in fact infinitesimal, $\varepsilon\delta(0) = 1$!) It is shown that for particular choices of W fusion-like solutions [L. de Broglie, *Théorie générale des particules à spin (méthode de fusion)*, Gauthier-Villars, Paris, 1943] exist which can describe particles of spin 0 and spin 1 with zero and with finite rest-masses. The scattering solutions of (2) are also investigated and it is shown that (2) does not give rise to any scattering between the particles. An attempt is made to give a physical interpretation to ψ_2 by using the conservation laws. However, no interpretation of the negative-energy solutions is given.

S. Schweber (Waltham, Mass.)

11927:

Mitra, A. N.; Saxena, R. P. Nucleon-antinucleon interaction in the New Tamm-Dancoff approximation. Proc. Roy. Soc. London Ser. A **257** (1960), 59–73.

The authors set out to analyse low-energy nucleon-antinucleon elastic scattering within the context of pseudoscalar meson theory. To this end a three-dimensional integral equation is written down, which takes account of the repeated effect of the long-range interaction due to exchange of one pion, and of the short-range interaction due to virtual annihilation into one pion. The integral equation is resolved into partial waves, and approximate solutions for the s and p scattering phases in the energy range 0–140 MeV are obtained by a variational method. The effects of real annihilation processes are not included in these calculations, although found to be very important in the work of Ball and Chew on the same problem [#11929]. {The reviewer found two persistent errors in sign in the elimination of the small components of the anti-nucleon wave function, and these appear to be carried over to the succeeding numerical treatment.}

G. R. Allcock (Liverpool)

11928:

Mitra, A. N.; Saxena, R. P. Renormalization for nucleon-antinucleon interaction. Proc. Roy. Soc. London Ser. A **257** (1960), 74–82.

This paper forms part of the work reviewed above [#11927]. The integral equation generates meson self-energy parts containing overlapping vertices. The self-energy and vertex parts are renormalized in closed form with the help of Ward's techniques, and are found to produce a large reduction in the magnitude of the virtual annihilation terms. The net effect of virtual annihilation is to give appreciable additive contributions to the scattering amplitude. These contributions are evaluated numerically, and found to be non-zero in all the s and p states of unit isobaric spin. The latter curious result may be related to the errors mentioned above.

G. R. Allcock (Liverpool)

11929a:

Ball, James S.; Chew, Geoffrey, F. Nucleon-antinucleon interaction at intermediate energies. Phys. Rev. (2) **109** (1958), 1385–1391.

11929b:

Ball, James S.; Fulco, Jose R. Nucleon-antinucleon scattering. Phys. Rev. (2) **113** (1959), 647–650.

A detailed theory of the intermediate energy scattering of an antinucleon by a nucleon is developed in the first of these papers. Calculations made with this theory are extended in the second paper, to include the elastic differential cross-sections, the charge-exchange differential cross-section, and the absorption cross-sections, at three energies in the range 50–260 MeV. The predictions of the theory are found to agree with the experimental data available. The results are further applied to establish the parameters of an optical model for antinucleon-nucleus scattering.

The nucleon-antinucleon interaction potential, from which the success of this work derives, comprises a one-pion exchange term, phenomenological terms of shorter range, and a totally absorbing boundary somewhere within a radius of one-third of a pion Compton wavelength. In the one-pion term due allowance is made for the difference in sign between the mesic changes of nucleon and antinucleon. The shorter range terms important in this problem are shown to be those which are dominated by two-pion exchange, and they are therefore taken over without change of sign from the phenomenological nucleon-nucleon potential. The problem is then one of solving the radial Schrödinger equations for the various s, p, d and f partial waves, and it is tackled with the help of the W.K.B. approximation. The effects of theoretical uncertainties in the interaction are discussed in the first paper, and an interesting and instructive comparison with nucleon-nucleon scattering is made there.

G. R. Allcock (Liverpool)

11930:

Tzou, Kuo-Hsien. Un modèle des leptons. Champs de l'électron-muon et du neutrino. C. R. Acad. Sci. Paris **251** (1960), 2659–2661.

11931:

Shimodaira, H.; Takahashi, Y. Identities between strong and weak interactions. Nuovo Cimento (10) **19** (1961), 537–545. (Italian summary)

11932:

Iwao, S. A theory of hyperfragments. I. Binding energies and level structure. Nuovo Cimento (10) **17** (1960), 491–515. (Italian summary)

This paper presents a method of analysis of hyperfragment binding energies (to be followed by part II presenting an analysis of decay dynamics) in terms of a few "low-energy parameters of the hyperon-nucleon interaction" which is patterned after the shell model calculations in ordinary nuclei. The emphasis is on the systematics rather than detailed use of plausible explicit potentials and approximate many-body wave functions for this potential. It is inferred that $^4He_1$ has spin zero and that consequently the kaon is pseudoscalar. This systematic approach enables the author to make several predictions as to the existence of stable hyperfragments and to estimate their binding energies. The use of shell-model methods for such low mass-number systems is highly unconventional, but it appears to this reviewer that, since one has not so far succeeded very well in fully describing the corresponding nuclei by many-body wave functions derived from explicit nuclear potentials, a systematic, though unconventional model like this, has a better chance of success.

E. C. G. Sudarshan (Rochester, N.Y.)

11933:

Tonin, M. On some compound models for the strong interactions. Nuovo Cimento (10) **18** (1960), 304–315. (Italian summary)

Consider a set of n identical Majorana fields obeying the standard anticommutation rules, and form their tensor products. If the Majorana fields have zero mass (finite mass), the anticommutation rules are invariant under an n-dimensional unitary (orthogonal) group. The problem of "compound models for strong interactions" is then posed (at least in its group-theoretic aspects: no real dynamical concepts are discussed here) in the following form: can one find a subgroup of this n-dimensional Lie group which is isomorphic to the (assumed) "fundamental symmetry group" of the "elementary" particles? Specifically, the author considers the "high-symmetry group" containing the three-dimensional isotopic spin, hypercharge and hypernumber rotations and the two-dimensional baryon number rotation. Two sets of such explicit realizations, their relation to some "models" previously presented, and the question of separate P and C invariance in these general schemes are discussed.

E. C. G. Sudarshan (Rochester, N.Y.)

11934:

Ansel'm, A. A. Solution of equation for meson-meson scattering amplitude in the asymptotic region. Ž. Èksper. Teoret. Fiz. **38** (1960), 1887–1890 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1356–1358.

Author's summary: "The set of integral equations deduced in references 1 and 2 for the meson-meson scattering amplitude is solved exactly. An integral equation for the amplitude which depends only on one variable is obtained as a direct mathematical corollary of this set."

S. Bludman (Berkeley, Calif.)

11935:

Feldman, G.; Matthews, P. T.; Salam, A. The inelastic

scattering of elementary particles. II. Nuovo Cimento (10) **16** (1960), 549–555. (Italian summary)

[For part I see Matthews and Salam, Nuovo Cimento (10) **13** (1959), 381–393; MR **21** #7031.]

Authors' summary: "An approximate scheme for calculating the inelastic scattering of elementary particles is proposed. The scheme fully incorporates the requirements of unitarity and partially includes causality. When applied to the simple problems of π-p and π-π scattering it reproduces very simply some well-known results."

P. W. Higgs (Edinburgh)

11936:

Thouless, D. J. Vibrational states of nuclei in the random phase approximation. Nuclear Phys. **22** (1961), 78–95.

From the author's summary: "Single-particle states excited from the ground-state of a nucleus are studied by a perturbation expansion of the Green's function. Results differing from a shell-model analysis are found, and it is shown to be always possible to choose an independent particle wave function which makes all collective modes stable in the random-phase approximation. It is suggested that the excited-state wave functions be normalized in an indefinite metric, for then different eigenfunctions are orthogonal. A variational principle is established which could be useful in the analysis of low-lying eigenstates. Spurious states connected with various symmetries are shown to separate out and to possess zero energy."

J. R. Klauder (Murray Hill, N.J.)

11937:

Baumann, K.; Thirring, W. The two-body problem in the extreme relativistic limit. Nuovo Cimento (10) **18** (1960), 357–367. (Italian summary)

The intention of this paper is to explore the possibilities of an improved version of a composite model for the pion, proposed originally by Fermi and Yang [Phys. Rev. (2) **76** (1949), 1739–1743]. Since this is an extreme relativistic problem, a Bethe-Salpeter equation is set up for particles with rest-mass zero interacting through the intermediary of a heavy boson. The ladder approximation is used and high momenta are suppressed by a modification of the propagator in order to produce convergence. Scalar and pseudoscalar solutions for the bound-state problem are then studied by finding approximate solutions for the integral equations which the spectral functions must satisfy. The conclusion is reached that the eigenvalue equation admits scalar and pseudoscalar bound-state solution of zero energy.

J. M. Jauch (Geneva)

11938:

Baumann, K.; Freund, P. G. O.; Thirring, W. On the Fermi-Yang theory of the pions. Nuovo Cimento (10) **18** (1960), 906–913. (Italian summary)

According to Fermi and Yang [reference in #11937] the pion is assumed to be a bound state of a nucleon and an antinucleon. The Bethe-Salpeter equation for two nucleons is solved approximately by the method presented in #11937, and the pion-nucleon coupling constant is calculated. The authors say that the constant is of the order unity, and that the pion is capable of strong interactions with the nucleon.

H. Wakita (Hiroshima)

11939:

Karpman, V. I. The microscopic theory of the Fermi liquid. Ž. Èksper. Teoret. Fiz. **39** (1960), 185–188 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 133–135.

A proof is given that a fundamental relation of the phenomenological Landau theory of Fermi liquids, $\varepsilon(p) = \delta E/\delta n(p)$, where $\varepsilon(p)$ is the quasi-particle energy, E the total energy, and $n(p)$ the quasi-particle occupation number, follows for normal systems from the microscopic theory. This relation reduces at the Fermi surface to the Hugenholtz-van Hove theorem. The proof uses a method of Klein and Prange [Phys. Rev. (2) **112** (1958), 994–1007; MR **21** #5465] and therefore, though basically correct, contains the same errors as does the work of these authors. These errors have been removed in later work, thus far unpublished.

A. Klein (Philadelphia, Pa.)

11940:

Valatin, J. G. Generalized Hartree-Fock method. Phys. Rev. (2) **122** (1961), 1012–1020.

A generalized Hartree-Fock variational principle is formulated to determine the single-particle states, their pairing, and the occupation-number distribution of Fermi systems. The formalism is designed to cover the interesting physical problems such as the Bardeen-Cooper-Schrieffer theory of superconductivity [Phys. Rev. (2) **108** (1957), 1175–1204; MR **20** #2196] and the theory of nuclei based on an analogous principle. These problems have been studied by the author and other people, essentially according to the same Hartree-Fock principle. The present work is only a very formal description of the general procedure.

The Hartree-Fock equations are written in terms of two matrices in the doubled space composed of one-particle wave functions φ and their complex conjugates φ^*, and the redundancy is then eliminated by means of subsidiary conditions. Also treated are the small oscillations about an equilibrium solution and the two-point Green's functions from the present point of view.

Y. Nambu (Chicago, Ill.)

11941:

Barragán, A. Tables of angular momenta and electric quadrupoles of the states associated with the quadrupole-quadrupole interaction in a harmonic oscillator. Rev. Mexicana Fis. **9** (1960), 129–147. (1 insert) (Spanish. English summary)

In this paper, the energy values are calculated and tabulated for a system of N particles with a quadrupole interaction, utilizing the theory of Bargmann and Moshinsky [#11942].

J. M. Jauch (Geneva)

11942:

Bargmann, V.; Moshinsky, M. Group theory of harmonic oscillators. I. The collective modes. Nuclear Phys. **18** (1960), 697–712.

A system of N particles moving in a common harmonics oscillator potential possesses symmetries over and above the usual rotational symmetries. The additional symmetry group is isomorphic to the unitary group of N-dimensions. The infinitesimal generators of these symmetry transformations furnish new integrals which

can be used for the classification of the stationary states of the system.

The discussion of such a system is carried through in this paper for spinless particles without mutual interaction. The main concern is with that aspect of the classification scheme that brings out the collective nature of the states.

In the last section a quadrupole-type interaction between the particles is introduced and it is shown that it can be expressed as an algebraic expression of some of the additional integrals discussed in the first part of the paper. The eigenvalues of this interaction can therefore be calculated and they give rise to rotational bands. The classification according to the new integrals corresponds therefore to the collective behaviour of the system.

J. M. Jauch (Geneva)

11943:

Wu, Tai Tsun. Some nonequilibrium properties of a Bose system of hard spheres at extremely low temperatures. J. Mathematical Phys. **2** (1961), 105–123.

Author's summary: "The pseudopotential method is used to study a special type of flow for a Bose system of hard spheres. In the first-order approximation, the wave function of the entire system is assumed to be the product of identical single-particle wave functions, which in general are time-dependent. Such a flow is necessarily irrotational, and the single-particle wave function satisfies a Schrödinger equation with a nonlinear self-coupling term. On the basis of this equation of motion, the following properties of the Bose system are discussed: the effect of the rigid wall, the moment of inertia, the compressional wave, and a type of 'vortex filament'. In the second-order approximation, the wave function of the system is expressed in terms of two functions such that one of them describes the single-particle state suitable for most of the particles while the other one describes the pair excitations. The much more complicated equations of motion are found, but in this approximation the flow is no longer strictly irrotational. The compressional waves are also studied in the second-order approximation."

R. M. May (Cambridge, Mass.)

11944:

Sackman, Jerome L. Steady creep bending of a nonhomogeneous beam. J. Aerospace Sci. **28** (1961), 11–14, 33.

The theory of plane stress is used to obtain the distribution of bending stresses in an infinitely long, thin, vertically loaded, incompressible linear viscous beam with a viscosity coefficient that varies exponentially through its depth. The results obtained are compared with the known results based on elementary beam theory.

A. M. Freudenthal (New York)

RELATIVITY

See also A11288, 11764, 11765, 11825, 11828, 11830, 11838.

11945:

Bastin, J. A. An extension of the Newtonian law of gravitation. Proc. Cambridge Philos. Soc. **56** (1960), 401–409.

The gravitational force between two particles, one at rest relative to an assumed sub-stratum, the other moving towards it with velocity v, is assumed to be the Newtonian one, modified by a factor $(c+v)^{1/2}(c-v)^{-1/2}$. This hypothesis is applied in a very simple cosmological model (two equal large point masses M a great distance R apart) in order to discuss its implications for, amongst other things, Mach's principle and the galactic recession. An interesting feature of the model is that the dimensionless constant $GMR^{-1}c^{-2}$ can be calculated for it (and has the value $\frac{1}{2}$).

C. W. Kilmister (London)

11946:

Rastall, Peter. A solution of the Einstein field equations. Canad. J. Phys. **38** (1960), 1661–1664.

The author obtains an exact solution of the Einstein gravitational field equations which is non-static or cylindrically symmetric. A possible application is briefly discussed.

M. Wyman (Edmonton, Alta.)

11947:

Uhlmann, Armin. Über den Begriff der Energie bei gekrümmter Raum-Zeit-Mannigfaltigkeit. Wiss. Z. Friedrich-Schiller-Univ. Jena **9** (1959/60), 459–489.

This paper represents to some extent a review of earlier work on the concept of energy-momentum in general relativity. As his original contribution the author suggests that the concept of energy (and of the other general dynamical quantities) be tied to the state of motion of the observer. If T^{ik} ($i, k = 0, \cdots, 3$) stands for the right-hand side of Einstein's field equations, i.e., the "matter" tensor, and if the unit time-like vector ξ_i represents the state of motion of a congruence of observers, then the author suggests as a defining equation for energy to be measured on a three-dimensional space-like hypersurface Σ the expression $E = \int |g|^{1/2}\xi_i T^{ik} d\Sigma_k$, which is manifestly coordinate-invariant. In general E will not be conserved unless the Riemann-Einstein manifold possesses special symmetry properties, e.g., if ξ_i is a Killing field. Alternatively, the author points out that if the requirement of normalization of the ξ_i-field is dropped, one can always find a scalar field α, at least locally, such that $\bar{E} = \int |g|^{1/2}\bar{\xi}_i T^{ik} d\Sigma_k$, $\bar{\xi}_i = \alpha\xi_i$, is conserved. From their defining equations it is clear that both E and $\bar{E}$ vanish in any domain in which the gravitational field has no sources. The author considers that no reasonable definition of energy ought to include contributions by the vacuum gravitational field, in view of the principle of equivalence (e.g., "empty elevator").

P. G. Bergmann (Syracuse, N.Y.)

11948:

Bonnor, W. B. The mass of a static charged sphere. Z. Physik **160** (1960), 59–65.

The author shows that the mass m of a static charged sphere in general relativity necessarily contains a positive contribution from its electrostatic energy, so that the parameters m and e in the usual exterior solution (the Riessner-Nordstrom metric) are not physically independent. A number of special models are presented in which the explicit dependence of m on e is given.

S. Deser (Waltham, Mass.)

11949:

Fihtengol'c, I. G. Coordinate conditions in the Einstein

theory of gravitation. Ž. Èksper. Teoret. Fiz. **39** (1960), 809–813 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 564–566.

The condition for the relative invariance of the gravitational Lagrangian for arbitrary coordinate transformations is found to reduce to equations that are linear with respect to $\mathfrak{g}^{\mu\nu}=g^{\mu\nu}\sqrt{-g}$. By showing that the Einstein-Infeld coordinate conditions do not reduce to equations linear in $\mathfrak{g}^{\mu\nu}$, the author concludes that these conditions cannot be obtained from the requirement of the relative invariance of the Lagrangian under some family of coordinate transformations.

The transformation $\rho=\rho(r)$ is then applied to the Schwarzschild solution for an isolated particle. The Einstein-Infeld conditions are then imposed on the potentials when the resulting metric is expressed in cartesian coordinates and a differential equation for ρ is obtained. It is found that this equation does not have a simple solution. *G. L. Clark* (London)

11950:

Marder, L. Gravitational waves in general relativity. V. An exact spherical wave. Proc. Roy. Soc. London Ser. A **261** (1961), 91–96.

[For part IV see Pirani, same Proc. **252** (1959), 96–101; MR **21** #5479.] A construction is given for an exact solution of Einstein's empty space-time field equations representing an axially symmetric pulse wave with the properties that (i) the wave front is a sphere, outside of which all space-time is flat; (ii) a finite, expanding line singularity forms the axial diameter of this sphere. Because of (i), the solution would need to be generalized before a physical source could be introduced.

The presence of radiation is confirmed by the fact that a great distance from the centre of the wave the Riemann tensor is asymptotically of Petrov's canonical type II (with vanishing scalar invariants).

D. W. Sciama (Ithaca, N.Y.)

11951:

Buchdahl, H. A. Non-linear Lagrangians and Palatini's device. Proc. Cambridge Philos. Soc. **56** (1960), 396–400.

The vacuum field equations of general relativity, (1) $R_{ab}=0$, may be derived from the variational principle, (2) $\delta\int R(-g)^{1/2}d^4x=0$, by subjecting the metric tensor g_{ab} to arbitrary variations which, with their first derivatives, vanish on the boundary of the region of integration. Here the Lagrangian $R=g^{ab}R_{ab}$ is the scalar curvature of Riemannian space-time, and $R_{ab}=2(\Gamma_a{}^c{}_{[c,b]}+\Gamma_a{}^d{}_{[c}\Gamma_{b]}{}^c{}_d)$ is the Ricci tensor, formed from the Christoffel symbols $\Gamma_a{}^c{}_b$ of the metric g_{ab}. This procedure is called g-variation.

Alternatively the functions g_{ab}, $\Gamma_a{}^c{}_b$ may be varied independently, with variations vanishing on the boundary. This is called P-variation (and referred to, conventionally, but without historical foundation, as "Palatini's device"). For the variational principle (2), P-variation yields the same field equations as g-variation, namely, (1). For other variational principles, g-variation and P-variation may yield quite different results; in particular, P-variation need not yield the relations between g_{ab} and $\Gamma_a{}^c{}_b$ which are necessary if the space-time is to be Riemannian, namely, that the $\Gamma_a{}^c{}_b$ be Christoffel symbols of the g_{ab}.

In this lucid and informative paper, conceptual objections to P-variation, and formal difficulties in its application, are discussed in detail, with special reference to Lagrangians quadratic in the components of the Riemann tensor. Some results of Stephenson [Nuovo Cimento (10) **9** (1958), 263–269; MR **20** #7571] are strengthened and in part corrected.

F. A. E. Pirani (London)

11952:

Stephenson, G. Quadratic Lagrangians and gauge-invariance in covariant field theories. Proc. Cambridge Philos. Soc. **56** (1960), 247–251.

The Lagrangians are $L=R^2$, $R_{ab}R^{ab}$, $R^a_{bcd}R_a^{bcd}$, where R^a_{bcd} is the Riemann tensor formed from a non-symmetric affine connection Γ^a_{bc}, and contractions are performed with a non-symmetric metric tensor g_{ab}. The variational principle is $\delta\int L(-g)^{1/2}\,d^4x$, the Γ's and the g's being varied independently (the difficulties to which this procedure may lead are discussed in review #11951). Gauge-invariance implies that these Lagrangians are unaffected by the transformation $g_{ab}\to\lambda g_{ab}$. Physical significance is not discussed. *F. A. E. Pirani* (London)

11953:

Lévy, Jack. Une version modifiée de la théorie du champ unifié d'Einstein. J. Phys. Radium **20** (1959), 747–750. (English summary)

Pour éviter les difficultés auxquelles se heurtent les théories usuelles du champ unifié, nous sommes conduits à introduire dans un lagrangien un terme quadratique par rapport au vecteur de torsion de la connexion affine. Cette introduction est justifiée par les résultats de la théorie de M. H. Eyraud et un schéma hydrodynamique de la théorie des électrons de Dirac.

J. Charles-Renaudie (Montpellier)

11954:

Hennequin, Françoise. Étude mathématique des approximations en relativité générale et en théorie unitaire de Jordan-Thiry. Bull. Sci. Com. Trav. Hist. Sci. **1** (1956), 2e partie, 73–154 (1957).

L'auteur recherche les solutions approchées des équations du champ en relativité générale (partie I) et en théorie unitaire de Jordan-Thiry (partie II). Les coordonnées soit choisies isothermes; les conditions d'isothermie sont physiquement interprétées. Les développements limités des potentiels sont effectués par rapport à la variable $1/c^2$ en supposant, dans l'espace à quatre dimensions, que les potentiels sont voisins des potentiels galiléens. La méthode employée permet le calcul—de proche en proche—des coefficients du développement à un ordre quelconque. L'étude des dimensions des coefficients calculés permet le choix des tenseurs à identifier avec les variables de champ. Ainsi, dans la théorie de Jordan-Thiry, les potentiels de gravitation diffèrent de ceux définis par Y. Thiry par un facteur scalaire, et conduisent au choix d'un ds^2 dans l'espace à quatre dimensions conforme à celui choisi par Y. Thiry; ceci étant en accord avec une interprétation de Jordan. Les équations du mouvement sont aussi étudiées et comparées aux équations classiques, pour les corps présentant la symétrie sphérique.

J. Charles-Renaudie (Montpellier)

11955:

Peres, Asher. Nonlinear electrodynamics in general relativity. Phys. Rev. (2) **122** (1961), 273–274.

In Riemannian space-time with metric g_{ab}, field equations for a (skew) electromagnetic tensor F_{ab} are derived from a Lagrangian which is an arbitrary function of two algebraic invariants of F_{ab}: $L=L(I, J)$, where $I=(-g)^{-1/2}\epsilon^{abcd}F_{ab}F_{cd}$, $J=g^{ac}g^{bd}F_{ab}F_{cd}$. The corresponding generalizations of the Einstein-Maxwell equations are obtained from $\delta\int(R+L)(-g)^{1/2}d^4x=0$, where R is the Ricci scalar, as usual. These equations are solved in the spherically symmetric static case, and the role played by charge in this theory is discussed.

F. A. E. Pirani (London)

ASTRONOMY

See also 11779.

11956:

Brown, Ernest W. ★An introductory treatise on the lunar theory. Dover Publications, Inc., New York, 1960. xvi+292 pp. $2.00.

This is a reprint of a well-known classical book first published by the Cambridge University Press in 1896. The chapter headings are as follows: (1) Force-functions; (2) Equations of motion; (3) Undisturbed elliptic motion; (4) Form of solution. First approximation; (5) Variation of arbitrary constants; (6) The disturbing function; (7) De Pontécoulant's method; (8) The constants and their interpretation; (9) The theory of Delaunay; (10) The method of Hansen; (11) Method with rectangular coordinates; (12) The principal methods; (13) Planetary and other disturbing influences.

E. Leimanis (Vancouver, B.C.)

11957:

Dibaĭ, È. A.; Cicin, F. A.; Šarov, A. S. The application of the virial theorem to the dynamics of star clusters. Astronom. Ž. **37** (1960), 659–664 (Russian); translated as Soviet Astronom. AJ **4** (1961), 629–633.

Lorsqu'on applique le théorème du "viriel" à un amas d'étoiles de configuration sphérique, on utilise un paramètre $\tilde{R}$ défini comme suite. Soit M la masse totale, G la constante de la gravitation et Ω le potentiel du système: $\tilde{R}=-GM^2/2\Omega$. On suppose d'autre part que R est du même ordre de grandeur que le rayon a de cet amas sphérique. L'auteur établit l'expression de $\tilde{R}$ en fonction de a et de la densité $\rho(r)$ d'une distribution continue de masses; une étude empirique de cette expression l'amène à conclure que le rapport $\tilde{R}/a$ est compris entre 0,3 et 0,6; et donc à rejeter comme accidentelle la prétendue vérification de la théorie dite du "temps de relaxation" basée sur l'ordre de grandeur de la dispersion des vitesses dans l'amas des Pléiades. Nous pensons que cette théorie aurait besoin d'être fondée avant d'être verifiée ou contredite.

F. Nahon (Marseille)

11958:

Biermann, L.; Davis, Leverett, Jr. Considerations bearing on the structure of the galaxy. Z. Astrophys. **51**, 19–31 (1960).

Les auteurs considèrent différents modèles du disque galactique et de son halo incluant à la fois les forces d'attraction newtonienne et les forces dûes aux champs magnétiques et aux rayons cosmiques. Ils proposent un critère pour éliminer des modèles plausibles mais physiquement impossibles; ce critère consiste à utiliser une généralisation du théorème du viriel dûe à Chandrasekhar et Fermi.

F. Nahon (Marseille)

11959:

van Albada, G. B. Formation and evolution of clusters of galaxies. Bull. Astronom. Inst. Netherlands **15** (1960/61), 165–198.

This paper describes the formation of clusters by gravitational instability in a perturbed homogeneous universe of galaxies which is undergoing cosmic expansion. The velocity distribution of the galaxies, and the number of them per unit volume, are assumed to be subject to large fluctuations of about $\frac{1}{10}$ throughout large regions of space. At first the moments of order three in the velocity distribution are assumed to vanish, but later in the paper the effect of third- and higher-order moments is considered.

Formation of clusters of the observed size requires that the average cosmic density be about ten times the visible density. A moderate amount of intergalactic radiation is necessary to prevent the formation of very large clusters. Given these conditions, reasonable agreement with observation is obtained for those models which expand from a singular state and in which $\dot{R}<0$ throughout.

The conclusions have to be modified for the Lemaître model (in which $\dot{R}$ changes sign). Less invisible matter is needed, but the model shows a tendency to form very large clusters, and this tendency cannot be overcome by the mechanism proposed for the other models.

The treatment throughout is that of Newtonian cosmology.

W. B. Bonnor (London)

11960:

Herrick, Charles E. On the computation of nearly parabolic two-body orbits. Astronom. J. **65** (1960), 386–388.

After a discussion of the failure of the eccentric anomaly as an intermediate variable for the computation of the time, t, in nearly-parabolic orbits, the following expression is derived:

$$kt=\frac{x}{1+\lambda x^2}+\frac{x^3}{1+e}\left(\frac{1}{1+\lambda x^2}-\frac{1}{3}\right)+\frac{x}{1+e}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(\lambda x^2)^n}{2n+3}.$$

Here k is a function of the mass of the central body and of the orbital elements; $x=\tan v/2$, $\lambda=(1-e)/(1+e)$, v being the true anomaly and e the eccentricity. The series for kt is shown to be rapidly convergent in the areas of interest and valid for elliptical, parabolic and hyperbolic orbits.

J. A. O'Keefe (Chevy Chase, Md.)

11961:

Yarov-Yarovoĭ, M. S. The explicit expression for first-order secular perturbations in terms of the orbit elements. Astronom. Ž. **37** (1960), 764–777 (Russian); translated as Soviet Astronom. AJ **4** (1961), 722–734.

From the author's summary: "Explicit expressions for first-order secular perturbations have been investigated by means of a general expansion (according to Newcomb's method) into a power series in eccentricity and sine of one half the relative inclination of the orbit. An analysis of the number of terms in these expressions showed that the secular perturbations can be calculated just as rapidly by

means of analytical formulae as by the Halphen-Goryachev method. However, the proposed method has the advantage that harmonic analysis does not have to be applied, and that it gives a possibility of estimating the remainder. Besides the perturbation function was expanded into series averaged over only the mean anomaly of the perturbing planet, and also according to the Hill-Delaunea scheme.

"As an example, the first-order secular perturbations of the elements of Ceres produced by the eight large planets were calculated. The numerical values of secular perturbations obtained are very close to those obtained by V. F. Prokurin and K. Merfield."

J. Moser (New Rochelle, N.Y.)

11962:

Kurth, Rudolf. Zur Masse-Leuchtkraft-Beziehung. Z. Astrophys. **50**, 253–257 (1960). (English summary)

Two asymptotic mass-luminosity relations, for stars of very small and very large masses, respectively, are proposed and are supported by dimensional arguments. They are compared with the empirical mass-luminosity relations. *G. C. McVittie* (Urbana, Ill.)

11963:

Kurth, Rudolf. Einige Bemerkungen über pulsierende Gaskugeln. Z. Astrophys. **50**, 258–268 (1960). (English summary)

The linearised equation governing the radial oscillations of a gas sphere is set up, and a method of integrating the equation is outlined. The symmetry of the pulsations, the length of their periods and the phase lag can be dealt with adequately by a linearised theory.

G. C. McVittie (Urbana, Ill.)

11964:

Ambartsumian, V. A. On the problem of the mechanism of the origin of stars in stellar associations. Rev. Mod. Phys. **30** (1958), 944–946.

11965:

Ambarcumyam, V. A.; Saakyan, G. S. The degenerate superdense gas of elementary particles. Astronom. Ž. **37** (1960), 193–209 (Russian. English summary); translated as Soviet Astronom. AJ **4**, 187–201.

Les auteurs étudient les propriétés des gaz dégénérés aux très hautes densités.

L'état du gaz est défini par une condition d'énergie minimum, qui permet de trouver la population du gaz, constitué d'électrons, mesons, protons, neutrons et hyperons. Les hyperons sont stables, car leur décomposition produirait des particules dont les états sont déjà occupés. Les auteurs montrent alors qu'à des densités croissantes apparaissent des phases successives: une phase proton électron pour un nombre de baryons $N < 0.77 \cdot 10^{31}$ cm^{-3}, une phase neutron pour $N < 4.65 \cdot 10^{38}$ cm^{-3} est suivie d'une phase à mesons μ^-, pour $N < 6.4 \cdot 10^{38}$ cm^{-3}; au-dessus de cette densité ($\rho = 1.12 \cdot 10^{15}$g cm^{-3}) commence une phase hyperons, avec apparition successive à des densités croissantes des particules suivantes: Σ^-, Λ, n^*, Σ^0, Ξ^-, p^*, Σ^+, Ξ^0, Π^-, ces derniers apparaissant pour $\rho = 1.44 \cdot 10^{17}$g cm^{-3}.

L'étude de l'équation d'état, compte tenu de notre ignorance relative des forces répulsives entre hyperons, ne peut faire l'objet d'une étude approfondie; il paraît également difficile de donner des conclusions pour des densités supérieures à $5 \cdot 10^{16}$g cm^{-3}, en raison de l'existence éventuelle d'hyperons de masse supérieure à celle des Ξ^-.

Cette étude annonce des travaux sur les états superdenses de la matière, en liaison avec les idées d'Ambarcumyan sur la formation des astres à partir de matière superdense [Conférence Solvay, Bruxelles, 1958, p. 241; *Nestacionarnye zvezdy*, pp. 9–16, Akad. Nauk Armyan. SSR, Erevan, 1957; #11964]. *E. Schatzman* (Paris)

11966:

Raju, P. K.; Talwar, H. S. Radial pulsation of a cylinder in a magnetic field. Z. Astrophys. **52** (1961), 31–36.

The authors consider radial pulsations of a self-gravitating cylinder of gas of infinite conductivity. The cylinder contains a twisted magnetic field with axial symmetry, and is surrounded by a toroidal field. The equilibrium density of the gas is assumed uniform. It is shown that the presence of the field increases the frequency of the pulsations. *F. D. Kahn* (Green Bank, W. Va.)

11967:

Goldsworthy, F. A. Ionization fronts in interstellar gas and the expansion of H II regions. Philos. Trans. Roy. Soc. London Ser. A **253** (1960/61), 277–300.

The gas in the H II regions surrounding early-type stars is hot and ionized. Elsewhere in the H I regions, it is cool and non-ionized. In the present paper the author discusses the growth of an H II region around a source of radiation, surrounded initially by cool, non-ionized gas. The H I–H II boundary, or ionization front, is treated as a discontinuity. Within the H II region the author allows for the effects of recombination and of ionization, and for the cooling due to radiation by impurity ions. In any given case the gas is shown to remain very nearly isothermal. Similarity solutions are then presented for cases with cylindrical and spherical symmetry.

Various configurations may occur. If the source is relatively bright, the ionization front advances supersonically into the H I region, and a shock follows it in the H II region. If the source is relatively dim, a shock precedes the ionization front which now advances subsonically into the gas ahead of it. It may also occur that there are, simultaneously, shocks ahead of and behind the ionization front. In this case the methods of the paper do not give a unique solution. To find this, one must appeal to Axford's results on ionization fronts [see following review]. *F. D. Kahn* (Green Bank, W. Va.)

11968:

Axford, W. I. Ionization fronts in interstellar gas: the structure of ionization fronts. Philos. Trans. Roy. Soc. London Ser. A **253** (1960/61), 301–333.

The structure of ionization fronts [see #11967] is discussed in detail. Once again ionization, recombination and radiative cooling are allowed for. Various types of front are described; they may be supersonic or subsonic ahead or behind. A front which is supersonic ahead and subsonic behind contains a shock. To construct a front which is subsonic ahead and supersonic behind requires a very careful adjustment of the rates of ionization and

cooling, or else the transition cannot be made. The final section of the paper contains some general remarks about detonations and deflagrations, and the applicability of the Chapman-Jouguet hypothesis.

F. D. Kahn (Green Bank, W. Va.)

11969:

Parker, E. N. The hydrodynamic treatment of the expanding solar corona. Astrophys. J. **132** (1960), 175–183.

The author [same J. **128** (1958), 664–676] had developed a theory of mass loss from solar corona, adopting a hydrodynamical approach which predicted a velocity of expansion of the order of 500 km/sec. Recently J. W. Chamberlain [ibid. **131** (1960), 47–56] pointed out that this expansion velocity is too high and is the outcome of the singular solution of the hydrodynamical equations adopted for the discussion of the problem. He also suggested that this problem should be studied through the classical evaporation theory of Stoney, Jeans and others.

In the present paper the author justifies the use of hydrodynamical equations on the basis of some numerical estimates of the mean free paths for the ions at different levels in the solar corona and in the interplanetary space. The author finds that the mean free paths are everywhere smaller than the characteristic scale of the system and hence the pressure tensor can be approximately taken to be isotropic. At the end, the author examines the evaporation theory and shows that in this theory the expansion velocity goes to zero at large distances from the sun due to the assumption that the evaporation takes place from the top of the static atmosphere. In fact, he proves that if evaporation from the solar corona occurs at all, it is from a level at which the hydrodynamic expansion is already supersonic.

P. L. Bhatnagar (Bangalore)

11970:

Nahon, Fernand. La théorie des valeurs extrêmes et l'estimation de la vitesse d'évasion des étoiles dans le plan galactique. C. R. Acad. Sci. Paris **251** (1960), 1963–1965.

The stellar velocities exceeding 65 kms/sec. are very unsymmetrically distributed according to the direction. Their velocity vectors totally avoid the longitudes around that of the galactic rotation, $l=55°$. Generally the velocity of escape is estimated as the largest velocity observed. In the paper considered two extreme values at the upper tail of the distribution are used.

It is supposed that we have an ordered sample of n velocities with the galactic longitude $l=55°$. The joint distribution of the values of ranks m and p counted from the largest value are considered. The frequency function of the velocities in the direction of the galactic rotation is supposed to be $F(x)=1-[(\omega-x)/b]^k$ for $x\leqq\omega$. The constant k is supposed to be known, and the constants ω and b are estimated from the observed values x and y of the observations of the ranks m and p. In this way only two observed values enter the formulae and the maximum likelihood estimates $\check{\omega}$ and $\check{b}$ are $\check{\omega}=(y-M_1x)/(1-M_1)$, $\check{b}=(y-x)/(L_1(1-M_1))$, where L_1 and M_1 are functions of k, m and p.

Finally, unbiased estimates of ω and b are derived. They are of the same form, but the functions L_1 and M_1 are replaced by other functions L and M of k, m and p.

E. Lyttkens (Uppsala)

11971:

Massonie, Jean-Philippe. Estimation de la vitesse d'évasion des étoiles dans le plan galactique; résultats. C. R. Acad. Sci. Paris **251** (1960), 1966–1968.

Author's summary: "Dans une Note précédente [#11970] M. Nahon a indiqué une méthode d'estimation de la vitesse d'évasion, à partir des valeurs extrêmes de la distribution des vitesses radiales. Nous précisons ici la distribution initiale et nous appliquons la méthode à plusieurs échantillons en fournissant un intervalle de confiance pour les différentes estimations."

GEOPHYSICS

See also 11503.

11972:

Gleeson, Thomas A. A statistical theory of meteorological measurements and predictions. J. Meteorol. **18** (1961), 192–198.

The author discusses first the errors due to the fact that the value of meteorological quantities on a map are generally interpolated from observations at neighbouring stations. In order to obtain a theory of these errors it is supposed that the stations are randomly distributed. The error is expressed as a product of a random variable, ε, having the range $-1\leqq\varepsilon\leqq+1$, the gradient of the meteorological quantity considered, and the linear size l of the unobserved region in the direction of this gradient. The mean and variance of l are expressed in terms of the mean and variance of the unobserved area, S, surrounding the point considered. The last-mentioned mean and variance, however, seem to have been underestimated, because it is supposed that S is smaller than the ratio, a, between the area of the region considered and the number of observation stations. However, assuming also that the standard deviation of ε is 1/6 of its range, the following formula is obtained for the squared mean error of an interpolated value $\hat{q}$ of a meteorological quantity: $\sigma_q^2=0.056a\left(\frac{\partial q}{\partial x}\right)^2$ where σ_q is the mean error of q, a the ratio between the total surface and the number of observation stations, and the partial derivative stands for the gradient of q.

After having computed mean errors of observed variables, the author proceeds to errors of functions of observed variables, and as an example the speed of a through line as given by Pettersen's formula is considered.

Errors in predictions of time-dependent variables are treated for the case where the time-dependence is approximately linear. In this case there exists an analogy to some problems in quantum mechanics.

E. Lyttkens (Uppsala)

11973:

Yoshiyama, Ryoichi. Stability of waves through a heterogeneous medium and apparent internal friction. Bull. Earthquake Res. Inst. Tokyo **38** (1960), 467–478. (Japanese summary)

An attempt has been made to explain theoretically the observed attenuation coefficient of seismic waves and their dependence on the wave periods. The problem has been further simplified by assuming that the property of the medium changes only in one direction and the wave

equation reduces to a one-dimensional one. The special case in which the density of the medium is constant and the structure varies periodically, leading to a wave-velocity distribution in the form of $(1+b\cos mx)$ in the direction of the x-axis, has been solved.

S. K. Chakrabarty (Howrah)

11974:

Lambert, Walter D. The gravity field of an ellipsoid of revolution as a level surface. Ann. Acad. Sci. Fenn. Ser. A III No. 57 (1961), 42 pp.

The gravity fields associated with different approximate possible shapes of the earth have been calculated in different co-ordinate systems. The theoretical calculations depend on some simplifying assumptions regarding the structure of the earth, which is not at all homogeneous nor does it have any regular symmetry. The calculated results thus differ appreciably from the simple theoretical ones. To avoid this, the author has introduced the concept of putting a thin layer of material of high negative density, so that correct mass of the earth may be obtained. The results obtained by the different methods have been compared. *S. K. Chakrabarty* (Howrah)

11975:

Braum, Franjo. Die numerisch-graphische gegenseitige Orientierung von Steilaufnahmen an den Stereogeräten I. Ordnung für beliebige Gelände- und Lückengestaltung. Deutsche Geodät. Kommiss. Bayer. Akad. Wiss. Reihe B. Angew. Geodäsie No. 72 (1961), 55 pp.

Gegenüber dem mechanisch-optischen Vorgang der gegenseitigen Orientierung von Bildpaaren für photogrammetrische Zwecke wird hier ein Beitrag zur numerischen Orientierung geliefert, bei der die erhaltenen Ergebnisse so interpretiert werden, daß sie durch eine graphische Konstruktion gewonnen werden können. Bei zunächst beliebiger Lage der Messpunkte und der Annahme von gebirgigen Gelände wird die Allgemeinlösung nach der Methode der kleinsten Quadrate herbeigeführt. Abschliessend werden gewisse Sonderfälle behandelt, bei denen die Messpunkte inselartig zu Gruppen gehäuft und an bestimmten Stellen des Bildes (peripher, bzw. in der Basisprojektion oder quer zu ihr) verteilt sind. Nach fehlertheoretischen Gesichtspunkten werden gewisse Vereinfachungen vorgenommen, und ihre Berechtigung wird im einzelnen nachgewiesen. *H. Wolf* (Bonn)

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A10920, A10923, A10995, A10996, 11438.

11976:

Канторович, Л. В. [Kantorovič, L. V.]. ★Экономический расчет наилучшего использования ресурсов [Economic calculation of the best use of resources]. Izdat. Akad. Nauk SSSR, Moscow, 1959. 344 pp. 13.80 r.

The author works out mathematical programming formulations of the problems of optimal resource allocation in a planned economy. He shows how given output targets can be achieved most efficiently, taking into account various kinds of constraints, such as limited capacities and scarce resources. The discussion, which is intended to familiarize Soviet economic planners and project makers with the usefulness of this mathematical approach, is mostly in terms of specific numerical examples of production, transportation, and investment planning. It is more of interest to the economist and student of Soviet affairs than to the mathematician. Mathematical statements of the problems discussed and the conditions of their optimal solutions, however, are provided in two appendixes, where the author investigates also the mathematical properties of the general case of optimal planning, which he labels the fundamental problem of production planning. The Leontief input-output technique is treated as a special, static case of the fundamental problem.

G. Rimlinger (Houston, Tex.)

11977:

Amoroso, Luigi. L'equilibrio dei sistemi economici. Ann. Mat. Pura Appl. (4) **50** (1960), 295–307.

The paper is dedicated to Antonio Signorini on the occasion of his seventieth birthday. This seems to have induced the author to restrict his discussion to such literature as must have constituted part of the septuagenarian's required reading as student—no doubt a fine, courteous gesture. The three sections are called "The classical doctrine", "The antithesis of the historical school", and "The dynamic synthesis". In the last of these, utility is expressed as a function of quantities bought and of their rate of change. This utility, discounted over the individual's time horizon, is maximised under the usual budget constraint, leading to dynamic equilibrium conditions of somewhat higher generality than the usual static ones. Profit maximisation is treated analogously; and the results are taken to show that in economics the principle of least effort plays a role equivalent to that of the principle of least action in classical mechanics.

G. Morton (London)

11978:

Debreu, Gerard. Topological methods in cardinal utility theory. Mathematical methods in the social sciences, 1959, pp. 16–26. Stanford Univ. Press, Stanford, Calif., 1960.

The paper presents three applications to utility theory of a topological theorem of Thomsen and Blaschke. Let a 2-cell be simply covered by each of three one-parameter families of curves in such a way that curves of different families meet in at most one point. This cell can then be mapped topologically into the plane so as to take the curve families into families of parallel straight lines, provided a certain simple geometric condition is satisfied which states that certain curvilinear hexagons must "close".

First application. Let S be a set of alternative prospects. Given a pair of prospects, a and b, the prospect ab denotes the "stochastic prospect" in which either a or b will occur with equal probability. If a subject orders the prospects ab in accordance with his preferences, the ordering is said to be compatible with a utility function u if u preserves the order and if $u(ab)=\frac{1}{2}(u(aa)+u(bb))$. Under certain standard topological assumptions a simple necessary and sufficient condition is given for an ordering to be compatible with a utility function.

Second application. Given a pair of "actions" a and b from a set S, it is known that a subject will choose a with probability $p(a,b)$ and b with probability $p(b,a)=1-p(a,b)$. The function p on $S\times S$ is said to be compatible with a utility function u on S provided $p(a,b)\geqq$

$p(c, d)$ if and only if $u(a)-u(b) \geq u(c)-u(d)$. A clearly necessary condition for the existence of u is that $p(a, b) \geq p(c, d)$ be equivalent to $p(a, c) \geq p(b, d)$. The condition is shown to be sufficient as well, provided a certain continuity condition is also assumed. {This latter condition seems somewhat unnatural to the reviewer.}

Third application (roughly). A subject orders according to his preference commodity bundles $x=(x_1, \cdots, x_n)$ with n components, and one seeks conditions under which this ordering is compatible with a utility function u such that $u(x)=\sum_{i=1}^{n} u_i(x_i)$. (In this case the components x_i of x are termed "independent".) A clearly necessary condition for the existence of u is that if certain components of x are held fixed, the ordering on the remaining components will be independent of the values assigned to the fixed ones. It is shown that if there are at least three "essential" components, then the above condition is also sufficient. The author notes that Samuelson proved this, assuming the original ordering was induced by a differentiable function. An ingenious use of the Thomsen-Blaschke result allows him to remove the differentiability hypothesis. *D. Gale* (Providence, R.I.)

11979:

Block, H. D.; Marschak, J. Random orderings and stochastic theories of responses. Contributions to probability and statistics, pp. 97–132. Stanford Univ. Press, Stanford, Calif., 1960.

This is an extensive study of the logical relations among various conditions describing consistency of responses, in particular, between certain testable and untestable conditions. The present consideration of axioms of stochastic consistency is better supported by experience than is that of absolute consistency of choices. The results are too lengthy to set forth here. A survey of related literature is interspersed with the results. {Reviewer's remark: The principle illustrated on the bottom of p. 107 was enunciated by Pitman in his work on "closest estimators", in Proc. Cambridge Philos. Soc. **33** (1937), 212–222.}

J. Kiefer (Ithaca, N.Y.)

11980:

Enrick, Norbert L. Variations flow analysis. Technometrics **2** (1960), 373–386.

11981:

Emanuel, H.; Klaassen, L. H.; Theil, H. On the interaction of purchasing motives and the optimal programming of their activation. A new approach to the evaluation of advertising themes. Management Sci. **7** (1960/61), 62–79.

For many reasons it would be important to know the effects of various motives upon purchasing decisions, and of their interaction. In their paper the authors define a method of analysis on the basis of the hypothesis that the introduction of a new motive has the same percentage effect on buyers and non-buyers regardless of whatever motives are already operative; thence they proceed to an outline of how information on buying motives may be derived from market research and used in finding the best allocation of advertisement expenditure. A hypothetical example of preferences between sea and air travel illustrates the method, and a useful large-sample test of the null hypothesis of balanced interaction is given. *G. Morton* (London)

11982:

Ackoff, Russell L. (Editor). ★Progress in Operations Research, Vol. I. Publications in Operations Research, No. 5. John Wiley & Sons, Inc., New York-London, 1961. xii+505 pp. $11.50.

A set of eleven expository articles, all of which will be reviewed separately.

11983:

Gould, A. H. Operational research, its methods and application: a review and prospect. J. Inst. Actuar. **86** (1960), 109–160.

A historical review of the development of operational research in England, from its beginnings in World War II to its adoption in industry. The types of theoretical techniques used and the kinds of problems studied are listed. A short forecast of future development is given.

P. M. Morse (Cambridge, Mass.)

11984:

Flagle, Charles D.; Huggins, William H.; Roy, Robert H. (Editors). ★Operations research and systems engineering. The Johns Hopkins Press, Baltimore, Md., 1960. x+889 pp. $14.50.

A compendium, representing the lectures in a two-weeks' course in the subject, given at Johns Hopkins. As usual with such collections, the material is spotty, with some quite "philosophical" articles at the beginning, trying to define the subject matter without getting below the surface, and some fairly solid sections later on, particularly those on flow-graphs and system stability, but even these are at a fairly elementary level. There are also "introductions" to inventory theory, queuing theory, linear (but not dynamic) programming, game theory and symbolic logic, plus three "case studies", one of them on newspaper publishing which is in sufficient detail to be of interest. This is another book designed to give administrators an idea of the scope and applicability of the subjects, but not detailed enough to be of much help to serious students. About par for such books.

P. M. Morse (Cambridge, Mass.)

11985:

Réczey, G. Methodik zur Ausarbeitung einer Mechanisierung der Bauindustrie, durch die optimale Leistungen erzielt werden. Acta Tech. Acad. Sci. Hungar. **31** (1960), 147–174. (English, French, and Russian summaries)

11986:

Newell, G. F. Queues for a fixed-cycle traffic light. Ann. Math. Statist. **31** (1960), 589–597.

The steady-state behavior of an automobile queue, with Poisson arrivals and with a cyclic red-green traffic light, is calculated. Analytic expressions for the average queue length and the average delay are obtained. The results are compared with those of earlier work on the same subject. *P. M. Morse* (Cambridge, Mass.)

11987:

Рейнфелд, Н. [Reinfeld, N. V.]; Фогель, У. [Vogel, W. R.]. ★Математическое программирование [Mathematical programming]. Translated from the English by G. N. Andrianov and B. N. Mihalevskiĭ; edited by A. A. Konyus. Izdat. Inostr. Lit., Moscow, 1960. 303 pp. 12.40 r.

The original [not reviewed in MR] was published by Prentice-Hall, Englewood Cliffs, N.J., 1958.

From the Russian preface: "This work deals with an active field in present-day applied mathematics, namely, linear programming, which raises the problems of business management to a more scientific level."

11988:

Arnoff, E. Leonard; Sengupta, S. Sankar. Mathematical programming. Progress in Operations Research, Vol. I, pp. 105–210. Wiley, New York, 1961.

The authors, in an almost comprehensive manner, report on developments in linear programming. Thus in this single chapter they consider computational aspects of the subject (e.g., the number of iterations), transportation problems, the assignment problem, the traveling-salesman problem, revised simplex procedures, dynamic linear programming (e.g., the caterer problem and production smoothing), sensitivity, parametric linear programming, integer solutions, stochastic programming and applications of linear programming. The chapter is more in the nature of a report than an exposition on the subjects treated, and hence many details are omitted.

T. L. Saaty (Silver Spring, Md.)

11989:

Pietrzykowski, Tomasz. An iteration method of linear programming. Prace ZAM Ser. A No. 5 (1960), 8 pp.

Consider the linear programming problem in vector notation which requires maximizing a linear function $c \cdot x$ where $c=(c_1, \cdots, c_n)$, $x=(x_1, \cdots, x_n)$, subject to the constraints $a_i x+b_i>0$ $(i=1, \cdots, m)$ and $x \geqq 0$. Let x be a vector for which at least one of the constraints is zero. Let $\delta \geqq 0$ be given and consider the indices $1 \leqq i_1 < \cdots < i_k \leqq m$ associated with x such that $a_{i_j} \cdot x+b_{i_j} \leqq \delta$ $(j=1, \cdots, k)$. Define $\alpha_i=-(a_i x+b_i)/a_i \cdot y(x, \delta)$ where $y(x, \delta)$ is a solution vector of the inequalities $(a_{i_1}, \cdots, a_{i_k}, c_i) \cdot y>0$ $(i=1, \cdots, m)$, if it exists; otherwise $\alpha_i=0$. Consider

$$f(x, \delta) = x+y(x, \delta) \min_i [\max(\alpha_i, 0)]$$

and suppose that x^0 and $\delta_0>0$ are given, where x^0 has the same properties as x. Define $\delta_{i+1}=\delta_i/2^p$ where p is the smallest positive integer such that $y(x^i, \delta_i/2^p) \neq 0$ (otherwise $\delta_{i+1}=0$) and where the iterations are given by $x^i=f(x^{i-1}, \delta_i)$. The author proves that as $i \to \infty$, x^i converges to a solution of the linear programming problem.

T. L. Saaty (Silver Spring, Md.)

11990:

Lemke, C. E. The constrained gradient method of linear programming. J. Soc. Indust. Appl. Math. **9** (1961), 1–17.

This paper describes an algorithm for finding the "constrained gradient" which arises in a method of linear programming given by J. K. Thurber. The approach used is to reduce the search for the gradient to a quadratic programming problem, which is then solved with an algorithm similar to one constructed by E. M. L. Beale.

J. J. Stone (Menlo Park, Calif.)

11991:

Wu, Chang-pu. The optimum operation for a yearly regulated water reservoir. Sci. Record (N.S.) **4** (1960), 108–110.

Summary of a research report in which the procedure of J. D. C. Little [Operations Res. **3** (1955), 187–197] for optimising the output of a hydro-electric scheme with prescribed side conditions, is generalised to include the further side condition that the probability of flooding above the reservoir is bounded above by a constant less than unity.

P. A. P. Moran (Oxford)

11992:

Hanssmann, Fred. A survey of inventory theory from the operations research viewpoint. Progress in Operations Research, Vol. I, pp. 65–104. Wiley, New York, 1961.

A description of the kinds of inventory systems which have been studied mathematically and a bibliography of the relevant papers on the subject.

P. M. Morse (Cambridge, Mass.)

11993:

Sakaguchi, Minoru. Strategic information and non-cooperative games. Kōdai Math. Sem. Rep. **12** (1960), 91–101.

Using three examples of continuous poker models, the author shows how the solution of a game is related to the information pattern of the game. The examples studied are two-person stud poker with a single bet, two-person draw poker, and three-person stud poker with a single bet. For these games, it is shown that the player who moves first and gives information about his hand is at a disadvantage.

M. Dresher (Pacific Palisades, Calif.)

BIOLOGY AND SOCIOLOGY

See also 11507, 11979, 12000.

11994:

Rashevsky, N. Some remarks on molecular biology. Bull. Math. Biophys. **22** (1960), 365–370.

Author's summary: "The binding energy of a very long molecular chain, composed of different classes of molecules, depends in general on the order of the molecules. It is shown that under very general conditions there exists for a given brutto chemical composition of a chain, a class of chains which is characterized by a total binding energy which is equal to the total binding energy of any other prescribed chain of different composition within the limits of unsharpness of the energy level. This establishes a criterion for mapping of a class of configurations of long chain molecules on another class. To the extent that a mapping constitutes a generalized code, those results contribute to the theory of molecular codes. Applying to our results the results of a previous paper [same Bull. **21** (1959), 309–326; MR **21** #7124], we arrive at the conclusion that the self-replication of a living molecule may be the property not of a particular structure but of classes of structures."

A. Rapoport (Ann Arbor, Mich.)

11995:

Nelder, J. A. A note on some growth patterns in a simple theoretical organism. Biometrics **17** (1961), 220–228.

11996:

Bartlett, M. S. Some stochastic models in ecology and epidemiology. Contributions to probability and statistics, pp. 89–96. Stanford Univ. Press, Stanford, Calif., 1960.

From the author's summary: "I consider how far a stationary distribution of population size in a certain class of models can be said to exist when there is an absorbing barrier at the extinction state. Provided the extinction probability is small enough, an effectively stationary process may be expected for quite a long time. A model for measles epidemics is considered, and predictions are compared with official statistics."

J. Kiefer (Ithaca, N.Y.)

11997:

Rashevsky, N. On the mutual influence of individuals in a society. Bull. Math. Biophys. **23** (1961), 15–18.

Models of social behavior based on interaction of influences exerted by various classes of individuals on each other has been discussed by the author in numerous previous publications. The principal features of these models were (1) a parameter of inherent inclination to perform some action (distributed in the population), (2) a propensity to imitate an action (depending on the number of people performing it), and (3) coefficients of influence (characteristic of various classes). Some results pertain to conditions under which the behavior of the bulk of the population is controlled by a given class. In the present paper it is shown that if the parameter of inherent inclination shifts in such a way that the behavior imposed by the "ruling" class becomes progressively less acceptable, i.e., the distribution of that parameter in the population becomes more and more skew toward the negative end, the condition of control will cease to be satisfied for the class in power and will pass to the class seeking to impose the other of the two assumed forms of behavior.

A. Rapoport (Ann Arbor, Mich.)

11998:

White, Harold. The formation of cell assemblies. Bull. Math. Biophys. **23** (1961), 43–53.

A "cell assembly" is a group of neurons in the central nervous system so interconnected that they are capable of maintaining activity by mutual stimulation. It is assumed by some neurophysiologists that groups of cells upon which simultaneous activity is imposed for prolonged periods become cell assemblies.

Suppose the net excitation of several groups of nerve cells is given by

$$\varepsilon_i = f_i + \theta_i n_i g_i - j \sum_{\nu \neq 1} n_\nu g_\nu, \tag{1}$$

where f_i is the input to the ith group, g_i the average output per cell, n_i the number of cells, θ_i an index of mutual excitation (assumed to increase as the simultaneous activity of the cells in the group continues), and j represents the strength of inhibition upon the cells of the ith group by the active cells of other groups.

Further, $g = \varphi(\varepsilon)$, where φ is a certain monotone increasing sigmoid-shaped function tending to a horizontal asymptote (derived elsewhere).

The author considers several special cases of (1) under various assumptions about previous history of two groups of cells ($i = 1, 2$) and about the targets of the outside input f. His conclusions concern the conditions under which each group is "ignited" and under which the two groups of cells coalesce into a cell assembly.

A. Rapoport (Ann Arbor, Mich.)

INFORMATION AND COMMUNICATION THEORY

See also A11094.

11999:

Kallianpur, Gopinath. On the amount of information contained in a σ-field. Contributions to probability and statistics, pp. 265–273. Stanford Univ. Press, Stanford, Calif., 1960.

Following Gel'fand, Kolmogorov, and Yaglom [Dokl. Akad. Nauk SSSR **111** (1956), 745–748; MR **18**, 859], the information of P_1 relative to P_2 (where the P_i's are probability measures) contained in a σ-field is defined as the supremum of the information contained in finite subfields. Properties of information are studied, representation and finiteness theorems being obtained which generalize those of the above reference for the information of one transformation relative to another.

J. Kiefer (Ithaca, N.Y.)

12000:

Luce, R. Duncan. The theory of selective information and some of its behavioral applications. Developments in mathematical psychology (edited by R. D. Luce), pp. 1–119, Free Press, Glencoe, Ill., 1961.

In the first section, the author presents an exposition of the discrete theory of information aimed primarily at mathematically literate behavioral scientists. That is to say, the principal aim of the exposition is to impart understanding of the meaning of the principal theorems and of the underlying assumptions, rather than to present the proofs in all their rigor. Where the purpose of greater understanding seems served, several lines of proof are indicated. Connections are also traced to multivariate theory and to relations between statistical tests and estimations of entropy, i.e., to the role played by the information concept in the statistical evaluation of patterns obtained in experimental data.

The second section is a review of applications of information theory in behavioral science, primarily psychological experiments inspired or evaluated by the use of this tool. Practically all the work examined falls between 1950 and 1955. The topics include estimates of redundancy of printed and spoken language, the interpretation of Zipf's law in terms of information theory, the estimates of the capacity of human beings viewed as channels, choice reaction time theory, information transmitted in absolute judgements, and applications of the information concept to problems of memory, conditioning, and perception.

The exposition includes in each case the statement of the theoretical problem, the results, and a critical discussion, in which opinions of several workers in the field and the authors are interspersed so as to exhibit the many facets and often the difficult and questionable aspects of this approach. Nowhere in behavioral science does the "amount of information" emerge with the same clarity and force as the classical theoretical constructs of physical science emerged in their time. Nevertheless, the

impression is gained that the introduction of this concept has been responsible for considerable progress and integration in several parts of quantitative psychology.

A. Rapoport (Ann Arbor, Mich.)

12001:

Beutler, Frederick J. Approximations to Wiener optimum filters and predictors. J. Soc. Indust. Appl. Math. **8** (1960), 662–679.

A stable, realizable, time-invariant filter is characterized by a weighting function $K(t)$ such that (*) $K(t)=0$, $t<0$, $\int_0^\infty |K(t)|dt<\infty$. The general problem which led to this paper is the practical question of the extent to which an operator corresponding to a stable lumped-parameter network (hence of type (*)) can be used to approximate a Wiener optimum filter. The Wiener filter is supposed to operate on a second-order stationary process with known absolutely continuous spectral density $\Phi(\omega)$. Goodness-of-approximation is measured by mean-squared error.

Two methods of approximation are considered: (A) direct approximation of the operator corresponding to the Wiener filter; and (B) prior approximation of $\Phi(\omega)$ by a rational spectral density $\Phi_n(\omega)$, and then use of the optimum filter corresponding to $\Phi_n(\omega)$. For method (A), the author shows (Theorem 2) that, if $\Phi(\omega)$ is bounded, arbitrarily good approximations of the Wiener filter can be obtained. For method (B), the sufficient conditions he states for arbitrarily good approximations are more complicated. Some of the results are extended to the case of filtration in the presence of a possibly correlated additive noise.

E. Reich (Aarhus)

12002:

Parthasarathy, K. R. On the integral representation of the rate of transmission of a stationary channel. Illinois J. Math. **5** (1961), 299–305.

The author gives a definition of rate of transmission for channels with infinite memory, which specializes to the usual definition for finite-memory channels. (This rate of transmission is a function of the channel and a stationary input measure; it is not a measure of code length.) The author shows that then the ergodic capacity is equal to the stationary capacity, and that the former is attained whenever the latter is. (The equivalence of the stationary and ergodic capacities for finite-memory channels was first proved by Carleson [Math. Scand. **6** (1958), 175–180; MR **21** #3281] and Caregradskiĭ [Teor. Veroyatnost. i Primemen. **3** (1958), 470–474; MR **22** #916], and then by Feinstein [Information and Control **2** (1959), 25–44; MR **22** #9344] and Breiman [Illinois J. Math. **4** (1960), 246–252; MR **22** #9347]. The problem was raised in the mistaken notion that its solution would furnish an upper bound on the length of codes.)

J. Wolfowitz (Ithaca, N.Y.)

12003:

Peterson, W. Wesley. ★Error-correcting codes. The M.I.T. Press, Cambridge, Mass.; John Wiley & Sons, Inc., New York-London; 1961. x+285 pp. $7.75.

While the theoretical capabilities of a single channel for the transmission of information may fairly be said to be well understood at present, practical means for achieving these capabilities are in general still unknown. In the last few years, however, an increasing amount of effort has been devoted to this problem, and much progress has been made. This book is the first to deal with the subject at any length. The completeness of its coverage may be judged by the chapter headings: (1) The coding problem; (2) Introduction to algebra; (3) Linear codes; (4) Error-correcting capabilities of linear codes; (5) Important linear codes; (6) Polynomial rings and Galois fields; (7) Linear switching circuits; (8) Cyclic codes; (9) Bose-Chaudhuri codes; (10) Cyclic codes for burst-error correction; (11) Other decoding methods; (12) Recurrent codes; (13) Codes for checking arithmetic operations. To the reader possessing a minimum knowledge of information theory, this book is entirely self-contained. The style is brisk, the organization of material excellent; to those with the slightest interest in coding theory, it cannot be too highly recommended.

A. Feinstein (Urbana, Ill.)

12004:

Reiger, Siegfried H. Codes for the correction of "clustered" errors. Trans. IRE **IT-6** (1960), 16–21.

The author investigates the properties of (systematic parity-check or group) error-correcting codes for which the correctable errors patterns include all of the single "bursts" or "clusters" of errors having length s or less. An inequality relation involving n (word length), k (number of information symbols), and s is obtained, and a method is presented by which the author is able to construct several of what he calls $(n, k)_s$ codes. One example, the $(11, 5)_3$, is worked out in some detail. The method, based on the "cycle-and-add" property of maximal-length linear recurring sequences modulo 2, allows one to select favorable parity-check sequences, and thereby construct the code. A simple decoding scheme is also indicated.

R. C. Titsworth (Pasadena, Calif.)

12005:

King, Gilbert W. Table look-up procedures in language processing. I. The raw text. IBM J. Res. Develop. **5** (1961), 86–92.

This paper makes a strong case for the use of table look-up procedures in a variety of non-numerical data-processing applications. Although the techniques described are based specifically on a photographic memory developed by the author and various past and present associates, some of the ideas in the paper are likely to find more general applications. While much of the introductory part of the paper recapitulates some well-known properties of automatic storage media and of access procedures, several new techniques, of which the table look-up method of address modification is one of the most interesting, are presented. The storage of a dictionary for use in automatic language translation is used as an illustration.

The effect of certain difficulties on ultimately satisfactory exploitation of these techniques is insufficiently explored. While the author points out "that dynamic dissection of words . . . can lead to errors", his suggested remedy, "to anticipate these peculiar cases, and enter the whole word", is satisfactory only if anticipation is possible and practicable. It is not at all obvious that, for the very large tables the author contemplates storing, anticipation is possible. Indeed, the difficulties mentioned have led, in some experiments with the storage device not mentioned in the paper, to the production of English correspondents

such as "horsedensator" and "thatpenturoj" for the Russian words "kondensator" and "temperaturoj". The presence or absence of these peculiar errors is a function of the current contents of the table, hence may vary from time to time. While the effect of such errors need not be serious, and while the remedy may work, no evidence is presented to the effect that a satisfactory solution has as yet been achieved.

In the matter of searching for long addresses through a splitting procedure and the use of prefixes, no mention is made of the procedure to be followed if not all parts can be matched. In this case, the information to be matched is no longer present in the input register. In connection with a similar problem arising in partial matching, it is mentioned that a subsequent paper in this series "will elaborate the use of partial matching and show how the skipped information can be retrieved"; presumably the same thing will be done for the case of long address mismatches.

In spite of these shortcomings, however, it is not unlikely that the proposed techniques will find a significant range of satisfactory applications; they therefore deserve careful study. *A. G. Oettinger* (Cambridge, Mass.)

SERVOMECHANISMS AND CONTROL

See also A10920, A11169, A11189, A11190, A11191, A11192, A11193, 11543, 11544.

12006:

Truxal, John G. Computers in automatic control systems. Proc. IRE **49** (1961), 305–312.

A survey article about which the the guest editor says, "One of the most significant applications of computer equipment is in automatic feedback control systems. J. G. Truxal has been asked to bring this field of activity into focus by identifying the types of computers employed in the role of active controller, establishing major types of control functions, and carrying the reader to the advanced control systems. . . ."

J. Hartmanis (Schenectady, N.Y.)

12007:

Aĭzerman, M. A.; Gantmaher, F. R. On the stability of the equilibrium positions for discontinuous systems. Prikl. Mat. Meh. **24** (1960), 283–293 (Russian); translated as J. Appl. Math. Mech. **24**, 406–421.

The stability of equilibrium positions of systems of the type $dx/dt = Ax + \lambda x_1/|x_1|$, where x is an n-dimensional vector, A is a constant square matrix, and λ a constant vector, has been investigated by several authors. In this paper the same problem for the more general case $dx/dt = f^{\pm}(x)$ is investigated, where $f^{+}(x)$ represents the motion on the "upper" side $(+)$ and $f^{-}(x)$ represents the motion on the "lower" side $(-)$, respectively, of a given surface $F(x) = 0$. The authors discuss particularly the stability of equilibrium positions which occur on the surface of discontinuity. The general behavior of trajectories when approaching, leaving or passing the discontinuity surface is described. In particular the behavior of trajectories near equilibrium points is discussed. With the assumption that the functions $f^{\pm}(x)$ are analytic in the neighborhood of equilibrium points, conditions for stable and unstable equilibrium points are established.

I. Flügge-Lotz (Stanford, Calif.)

12008:

West, J. C.; Douce, J. L.; Leary, B. G. Frequency spectrum distortion of random signals in non-linear feedback systems. Proc. Inst. Elec. Engrs. C **108** (1961), 259–264.

An interesting and well-done engineering application of the known theory concerning the output spectrum following a zero-memory non-linearity for stationary Gaussian input, to provide approximate estimates of the (frequency) distortion produced by torque saturation in an otherwise linear control system, when responding to a random input signal.

A. V. Balakrishnan (Los Angeles, Calif.)

12009:

Pugachev, V. S. Statistical theory of systems reducible to linear. IRE Trans. **CT-7** (1960), 506–513.

It has been recognized for some time that the optimal (in the mean, square sense) estimator is hard to determine in general. However, if the functional form of the estimator is assumed then the problem reduces to an essentially linear one. This is the case that the author calls 'reducible to linear' (even though of course the operations on the input data are not necessarily linear), and in this paper derives and examines the associated integral equations. Specifically let $x(\sigma)$, $\sigma \in T$, be the observed stochastic process. Let $X(t)$ denote the n-dimensional function $(x(t_1), x(t_2), \cdots, x(t_n))$, $t_i \in T$, and let $d|t|$ denote the n-dimensional volume element. As estimators of a random variable $W(s)$, non-linear functionals of the form

$$f(X) = \int \sum_{1}^{N} g_k(s, t)\varphi_k(X(t), t, s)\, d|t|$$

are considered. In the first class of problems, the φ_k are fixed and the optimal (in the mean square sense) $g_k(s, \cdot)$ are to be determined. In the second type of problem, a functional of the form $f(X) = \int \varphi(X(t), t, s)d|t|$ for fixed n is assumed for the estimators, and the resulting integral equation is examined for the case where $W(s) = \int \theta(Z(t), t, s)d|t|$ similar in form to $f(X)$, which has been previously studied by Zadeh [J. Appl. Phys. **24** (1953), 396–404; MR **15**, 376]. The author shows that the kernel is of the positive type and indicates a method of solution using a Mercer series expansion.

{Some of the terminology of the paper is non-standard. Thus the term 'operator' is used for 'functional' and $AX(t)$ does not mean AX evaluated at t. The use of the theory of positive operators or quadratic forms on a function space would help considerably to condense the notation and results, as well as clarify the nature of the solutions of the integral equations. In view of the known practical difficulties associated with the theory, namely, the restriction to preassigned functional forms and the difficulty of obtaining the requisite statistics, some applications would have been most welcome.}

A. V. Balakrishnan (Los Angeles, Calif.)

12010:

Popovič, Konstantin P. [Popovici, C. P.]. On the stabilization condition for the sequence of positions of relay contacts for a given input sequence. Rev. Math. Pures Appl. **2** (1957), 509–534. (Russian)

This relates to the theory of idealized sequential relay circuits. Such a circuit is constituted as follows: There are m input elements (buttons, keys, etc.), n ordinary idealized relays, and p output elements (lamps). Each of these

elements is capable of two states, an on-state 1 and an off-state 0; so that the state of the circuit at any instant is given by three vectors, an m-dimensional relay input vector (upravlenie) K, an n-dimensional relay position vector (położenie) P, and a p-dimensional output vector E. The operation is by discrete time intervals, called takts, such that these vectors take fixed values K_k, P_k, E_k, in the kth takt, and at least one changes from one takt to the next. Thus the operation of the circuit is given by a sequence of states

$$(1) \qquad (K_k, P_k, E_k), \qquad k = 0, 1, 2, \cdots;$$

further, given K_k and P_k, E_k and P_{k+1} are uniquely determined. Associated with (1) is another sequence,

$$(2) \qquad (K_j, E_j), \qquad j = 0, 1, 2, \cdots;$$

it is formed from the previous one by omitting the vector P and then striking out successive repetitions of the same pair; and it represents the work of the circuit as seen from outside. The paper is concerned with the problem of designing a circuit when (2) is given but not the relay structure. It is supposed that this can be done in some way (subject to a condition that the K_k are not changed too rapidly); the thesis is to show under what circumstances the design can be modified so as to satisfy a "stabilization condition", viz., that there be in (1) at most two successive takts with the same K, and that the second (also the first if there is only one) of these remains unchanged until there is a change in K. It is shown that for this to be possible the sequence (2) must satisfy an analogous condition; further, in the case of two successive takts with the same K, the first one lasts only the time for one takt in (1). If this condition is satisfied, then the author shows that the stabilization condition can be satisfied without increasing its number of relays. The method is simply to show that superfluous contacts can be eliminated. {The reviewer found the paper obscure (possibly for linguistic reasons); and it seems that there is some error, not affecting the main conclusions of the paper, in the conditions on the initial takt ($k=0$) on p. 512.} A general account of the researches of which this paper forms a part, with reference to this and related papers, is given in French by G. C. Moisil [An. Univ. "C. I. Parhon" Bucureşti. Ser. Acta Logica **2** (1959), no. 1, 145–199; MR **22** #10878].

H. B. Curry (University Park)

12011:

Moisil, Gr. C. La théorie algébrique des circuits à éléments électroniques. I. Le fonctionnement séquentiel du circuit d'Eccles-Jordan. Com. Acad. R. P. Romîne **9** (1959), 755–757. (Romanian. Russian and French summaries)

The behavior of an ordinary symmetrical flip-flop built out of vacuum tubes is described. This is a simple example of a sequential circuit. A two-valued table of inputs, outputs, and the states of various leads is given, as well as equations describing this table in terms of Boolean algebra.

E. F. Moore (Murray Hill, N.J.)

12012:

Moisil, Gr. C. La théorie algébrique des circuits à éléments électroniques. II. Le cryotron à enroulements différentiels. Com. Acad. R. P. Romîne **9** (1959), 871–873. (Romanian. Russian and French summaries)

This paper suggests a Boolean-algebra method for treatment of several series-parallel networks made of cryotrons which have two windings used in opposite senses.

E. F. Moore (Murray Hill, N.J.)

12013:

Dionisio, J. J. The definition of entropy in the calculus of probabilities. Gaz. Mat. (Lisboa) **20** (1959), no. 74/75, 1–7. (Portuguese)

An exposition of parts of information theory.

12014:

Holland, John H. Cycles in logical nets. J. Franklin Inst. **270** (1960), 204–226.

The author describes a series of results relating to the problem of simulating arbitrary finite state machines by finite-state machines with a uniform bound on the length of their longest cycle. It is conjectured, but not proved, that such simulation is impossible. The conjecture is proved false for a class of machines called (n, r) cycles, and the conjecture is refined to include a uniform bound on both n and r, where r is the maximum number of feedback lines from points in the cycle to any switch in the cycle.

D. Arden (Cambridge, Mass.)

12015:

Aĭzerman, M. A.; Gusev, L. A.; Rozonoèr, L. I.; Smirnova, I. M.; Tal', A. A. Finite automata. I, II. Avtomat. i Telemeh. **21** (1960), 224–236, 359–368 (Russian. English summary); translated as Automat. Remote Control **21**, 156–163, 248–254.

A survey of fifteen American, one French, and nine Russian references on sequential circuits and sequential machines is given. The authors confine their discussion to synchronous machines, i.e., to assuming that time occurs only in integral values. Of the many terms (nonprimitive circuits, generalized neuron nets, etc.) such machines have been called by in the different papers, the authors prefer to call them finite automata. Many of the concepts arising in the different approaches are discussed in formal terms, including decompositions, isomorphisms of machines, and recording of inputs, internal states, and outputs. The idea of synthesizing an automaton operating in one time scale out of elementary components operating in a faster time scale is discussed in some detail.

E. F. Moore (Murray Hill, N.J.)

12016:

Raney, George N. Sequential functions. J. Assoc. Comput. Mach. **5** (1958), 177–180.

The theory of sequential machines is reformulated in this paper, to avoid the necessity of defining a sequential machine by an arbitrary table defined in terms of states. Instead, the function which maps input sequences into output sequences is assumed given, and from this the states of the sequential function are defined. If the number of these is finite, the sequential function does correspond to a sequential machine with a specified initial state. The paper indicates how to prove most of the theorems of the reviewer [*Automata studies*, pp. 129–153, Princeton Univ. Press, Princeton, N.J., 1956; MR **17**, 1140] in terms of this formulation.

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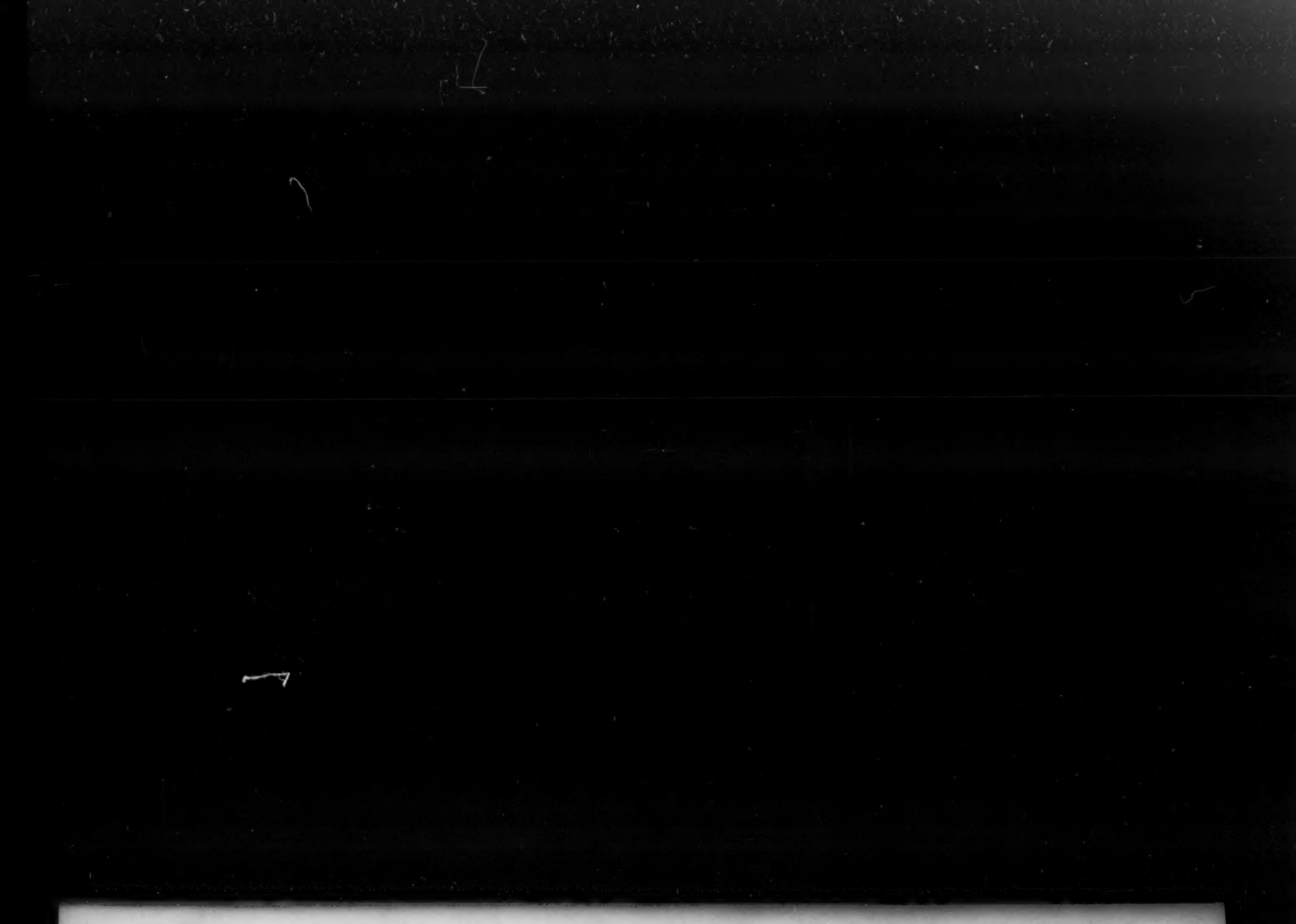
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